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A MODEL FOR PREDICTING SOIL

TEMPERATURE FROM AIR TEMPERATURE

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ABSTRACT

The prediction of soil temperature at the 1-inch depth from air temperature was accomplished by using a linear model developed from a Fourier series analysis of existing data. Values for soil temperature below the 1-inch depth were predicted by a one-dimensional heat flow equation.

INTRODUCTION

Both soil and air temperature are dynamic in nature, having a yearly cyclic appearance due to seasonal changes and having a diurnal variation of a periodic nature on a daily basis. These temperatures also change with respect to the depth below the ground surface and the height above ground level, respectively. Several reports have been published which have used Fourier series to describe the cyclic effect of the data on a yearly or daily basis (Penrod, Elliott, and Brown, 1960; Carson, 1961; Carson, 1963). Many reports have been published which use the one-dimensional heat flow equation in the prediction of soil temperature with depth, using soil surface temperature measurements (Langbein, 1949; Fluker, 1958; de Vries, 1958; Penrod et al., 1960; Van Wijk, 1963; Roberts, 1967; Wierenga, Nielsen, and Hagen, 1969; Hanks, Austin, and Ordrechan, 1971). Bonham and Fye (1970) used a linear regression analysis to estimate wintertime soil temperatures, utilizing daily mean air temperature as the independent variable. The papers listed above constitute only a portion of the publications which have been presented to analyze soil temperature.

SOIL TEMPERATURE ANALYSIS

Soil temperature and air temperature are both closely associated with many types of ecological problems. A few of these problems include the pre- diction of insect hibernation and hatch in the soil, seed germination, and plant growth. It is relatively expensive and very difficult to monitor air and soil temperatures in all locations where this type of information would be useful to the ecologist. Air temperature by itself, however, is relatively inexpensive and easy to measure. Also, air temperatures are already monitored

and recorded at a large number of locations throughout a state and the nation. Since air temperature is such an easy quantity to measure compared to other quantities which affect the soil temperature, a mathematical model for predicting soil temperature at various depths in the soil profile, using air temperature measurements as the driving mechanism, was studied.

The mathematical model which was developed to predict soil temperature was assumed to be a linear function between the air temperature and soil temperature at the 1-inch depth of soil (Fig. 1). The assumption that the air temperature is the main driving mechanism for the soil temperature is questionable. Other quantities such as solar radiation, the type of vegetative cover, and humidity and water content of the soil are believed to be important variables which affect the soil temperature also. However, these quantities are much more difficult to measure and have only recently been recorded (and then at only a very few sites). For this reason, it was felt that if air temperature could be used as a predictor for soil temperature, the method could be applied to areas in which the only measured quantity is air temperature.

Using the model shown in Fig. 1, the following equation was used to describe the relationship between the input and output of the model:

$$ST(t) = C \cdot AT(t) \tag{1}$$

where $ST(t)$ is the soil temperature as a function of time and $C \cdot AT(t)$ is the convolution (C) of the air temperature as a function of time ($AT(t)$). The convolution (C) is used as the linearizing mechanism between the air and soil temperature. Equation (1) can be expressed in integral form as

AIR TEMPERATURE

(INPUT)
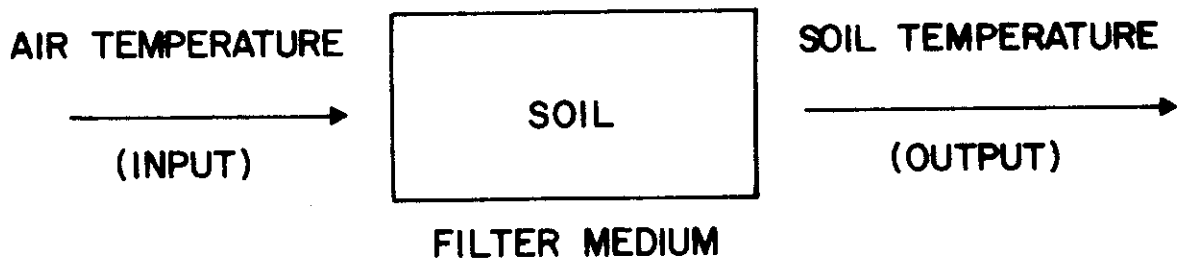
SOIL

FILTER MEDIUM

SOIL TEMPERATURE

(OUTPUT)

Fig. 1.   The assumed model between the air temperature (input) and soil
temperature at the 1-inch depth (output).

$$ST(t) = \int_{-\infty}^{t} C(t-\tau) \, AT(\tau) \, d\tau \tag{2}$$

where τ is the time lagging factor or the effect of the previous day's air temperatures on the relationship between the air and soil temperatures. In order to obtain the values of the convolution (C), the previous history of measured soil temperatures and air temperatures for a given site must be known. The convolution can then be determined and equation (1) can be used to predict soil temperature given the value of air temperature.

Equation (1) can be solved to determine the convolution by taking the Fourier transform of equation (1):

$$F(ST(t)) = F(C * AT(t))$$

where F( ) stands for the Fourier transform of the quantity in brackets. The above equation may be written as:

$$F(ST(t)) = F(C) \cdot F(AT(t)) \tag{3}$$

where the dot implies multiplication. Rearranging equation (3), one obtains:

$$F(C) = \frac{F(ST(t))}{F(AT(t))} \tag{4}$$

Equation (4) states that if a Fourier transform of the soil temperature and air temperature can be found, that division of the Fourier transforms will yield the Fourier transform of the convolution. The inverse transform of the Fourier transform of the convolution will yield the convolution (C). Substituting the convolution into equation (2), the soil temperature at any time (t) can be found.

Once the soil temperature at the 1-inch depth is determined, the one-dimensional heat flow equation as a function of time and depth is used to model the soil temperature as a function of depth in the soil layer. The one-dimensional heat flow equation states that:

$$C \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left(\lambda \frac{\partial T}{\partial z} \right) \tag{5}$$

where C is the volumetric heat capacity, λ is the thermal conductivity, T is the soil temperature, t is the time, and z is the soil depth. If the assumption is made that the soil medium is homogeneous and isotropic throughout, equation (5) becomes:

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial z^2} \tag{6}$$

where $a = \lambda/C$, and a is called the thermal diffusivity.

The use of the mathematical technique known as separation of variables will result in a solution of equation (6). The general solution is:

$$T(z,t) = (A \cos kt + B \sin kt) \cdot \exp(-k/a \; z)$$

where A, B, and k are coefficients of the solution. The coefficients in the equation above are evaluated from the boundary conditions of the problem. Using the boundary conditions stated in Van Wijk (1963) for periodic temperature variations in homogeneous and isotropic soils, which includes the soil temperature at the 1-inch depth as one of the boundary conditions, the solution of the above equation is:

$$T(z,t) = \bar{T} + \sum_{n=1}^{\infty} TA_n \exp(-zn^{\frac{1}{2}}/D) \sin (n\omega t + TO_n - zn^{\frac{1}{2}}/D) \qquad (7)$$

where $\bar{T}$ is the average soil temperature for the soil profile and is assumed to be the same at all depths (Wierenga et al., 1969), TA is the amplitude of the surface or uppermost temperature wave, and ω is the radial frequency. The constant TO is selected in order to make the term $\sin(n\omega t + TO_n - zn^{\frac{1}{2}}/D)$ = +1 when the surface or uppermost temperature is at its maximum value. D is called the damping depth and is a measure of the heat transfer which occurs.

Equation (7) shows that the soil temperature at any time and depth is equal to the average soil temperature plus a portion of the amplitude variation which occurs with time and depth through the exponential and sine functions. The constants $\bar{T}$, TA, D, ω, and TO in equation (7) are obtained by analysis of existing soil temperature data at a given site. Once the values of the coefficients have been determined for the site, equation (7) can be used to predict soil temperatures at different depths throughout the soil profile.

At Archer, Wyoming (Fig. 2), the University of Wyoming maintains a climatological station which measures maximum and minimum air and soil temperatures on a daily basis. The soil temperatures are measured from a minimum depth of 1 inch to a maximum depth of 72 inches below ground surface. The soil at the Archer Site is classified as an Ascalon series soil and contains gravelly material and clays plus silts in part of the soil profile. The ground cover is crested wheatgrass. The site is located in section 23 of township 13N in range 65W of Laramie County, Wyoming. The Archer climatological station is located very near the experimental grassland Pawnee Site and has a ground cover very similar to the Pawnee Site. Since the records of soil and air

Fig. 2. Climatological station at Archer, Wyoming.

temperature at the Pawnee Site are just beginning to be recorded, the data collected at the Archer Site is being used as a beginning basis for modeling the soil temperature at Pawnee. The results obtained with the Archer data will then be used together with the data obtained at the Pawnee Site to model the soil profile on both a yearly and daily basis.

The climatological site at Archer has been recording soil temperature data (maximum and minimum values) since September 5, 1963, on a daily basis. Air temperature data has been recorded for a much longer period. Since approximately 7 years of soil temperature data are available for analysis, 4 years of the data were picked at random to generate the soil temperature model. The other 3 years of data were then used to test the model.

The maximum and minimum soil and air temperature values on a daily basis were averaged together to obtain the mean temperature for each day. This was done for the 4 years of record. The daily mean temperatures were then smoothed on a yearly basis. The smoothing technique used was a 10-day moving straight average. Smoothing of the data was performed in order to remove extraneous effects from the data which would be considered to have an effect on prediction of the soil temperature from the air temperature. Other techniques of smoothing were tried, but it was found that a 10-day moving straight average gave the most desirable smoothing effect upon the data. Fig. 3 shows the smoothed values of the soil temperature at the 1-inch depth and the air temperature at Archer for the year 1969. The soil temperature values at 2.25 inches, 4.0 inches, and 8.0 inches were also smoothed in the same manner.

Fourier series analysis (Conrad and Pollak, 1962) was then applied to the soil temperature data at the 1-inch depth and the air temperature data
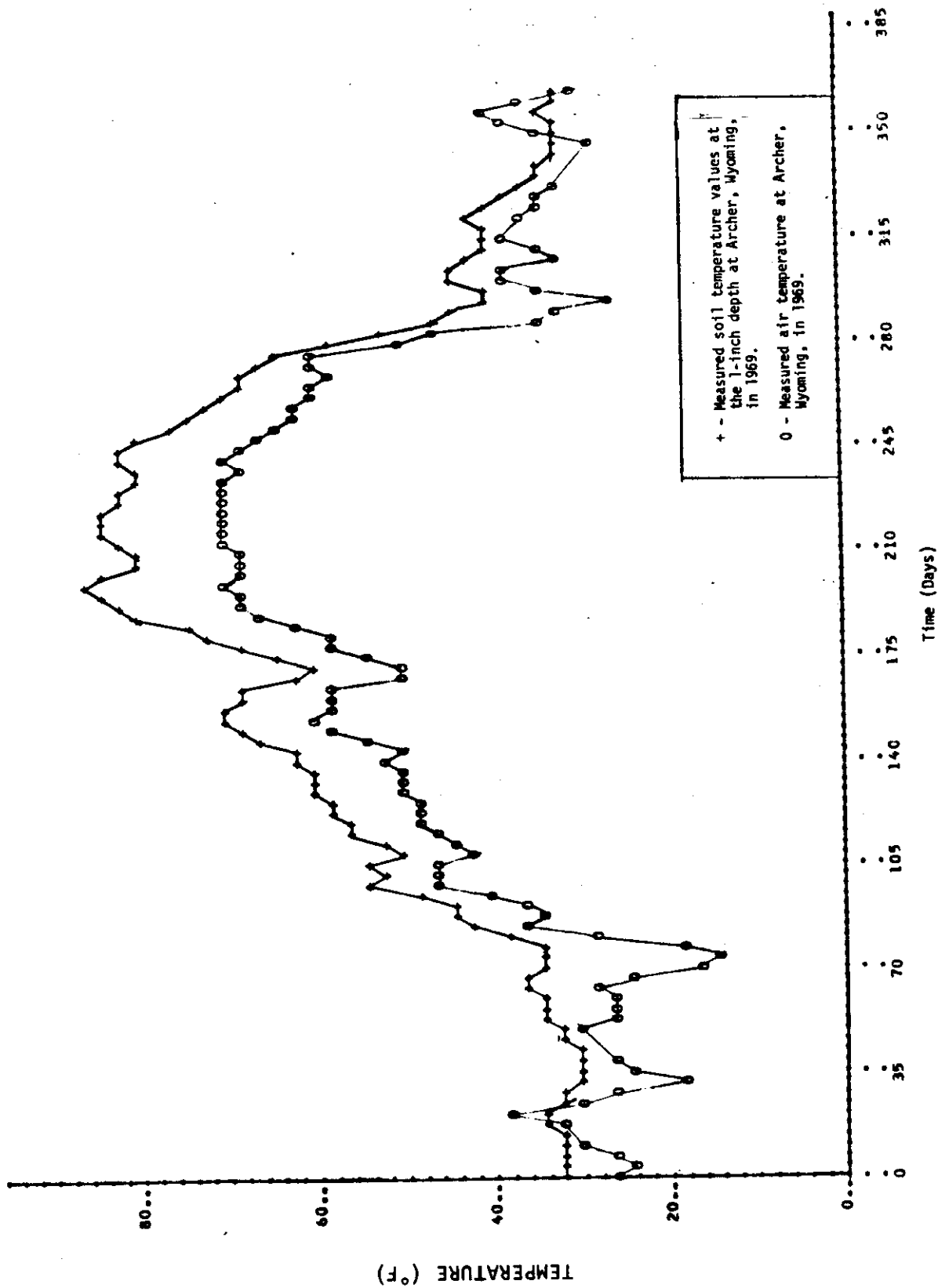
Fig. 3. Measured values of soil temperature and ambient air temperature at the 1-inch depth smoothed with a 10-day moving straight average.

on a yearly basis for the 4 years. The forms of the Fourier series for the
soil and air temperatures on a yearly basis are:

$$ST(t) = A_o + \sum_{n=1}^{\infty} (P_n \cos nt + q_n \sin nt) \tag{8}$$

and

$$AT(t) = P_o + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \cos nt) \tag{9}$$

respectively, where A_o, P_o, P_n, q_n, a_n, and b_n are all Fourier series coef-
ficients which are used to describe the soil temperature and air temperature
curves. There were 20 coefficients each used to fit the Fourier series to
the soil and air temperature curves.

Once the Fourier series for the air temperature and soil temperature
were found for each of the 4 years of record, the Fourier transforms of each
year's Fourier series were found. The Fourier transform of a Fourier series
turns out to be the values of the coefficients of the Fourier series. There-
fore, the Fourier transforms of equations (8) and (9) are:

$$F(ST(t)) = \frac{P_n}{2} - i \frac{q_n}{2} \tag{10}$$

and

$$F(AT(t)) = \frac{a_n}{2} - i \frac{b_n}{2} \tag{11}$$

By substituting the expressions obtained in equations (10) and (11) into
equation (4), the Fourier transform of the convolution is obtained. The
expression obtained is:

$$F(C) = \frac{a_n p_n + b_n q_n}{a_n^2 + b_n^2} + i \frac{(p_n b_n - a_n q_n)}{a_n^2 + b_n^2} \tag{12}$$

Now, δ_n will be used to represent the real part of equation (12) and μ_n will be used to represent the imaginary part of equation (12). Equation (12) may then be written as:

$$F(C) = \delta_n + i \, \mu_n. \tag{13}$$

By taking the inverse Fourier transform of equation (13), the equation of the convolution is obtained:

$$C(t-\tau) = \delta_0 + \sum_{n=1}^{\infty} \delta_n \cos n(t-\tau) + \mu_n \sin n(t-\tau) \tag{14}$$

where δ_0 is equal to A_0/P_0. A_0 is the average soil temperature, and P_0 is the average air temperature for the entire year's record which was used.

The values of δ_0, δ_n, and μ_n were determined for each of the 4 years of record used in generating the model. Some 20 values of δ_n and μ_n were calculated. The values obtained for each year's record for δ_0, δ_n, and μ_n were then averaged, and the values obtained were used as the coefficients in equation (14). Values of the convolution were then obtained from equation (14). It was found that only the first three coefficients generated in the convolution should be used in the model.

The model for the soil temperature at the 1-inch depth as a linear function of the air temperature has now been determined from the 4 years of record (see equation (2)). Using the model, the soil temperatures at the 1-inch depth were computed for the 3 years of record not used in generating

the model. The values were obtained by performing a numerical integration, Simpson's rule, on equation (2). The calculated values obtained from the model for the year 1969 (at Archer) which was not a year used in obtaining the coefficients are shown on Fig. 4, along with the actual values measured during that same year after the actual values were smoothed using a 10-day moving straight average.

By applying Fourier series analysis to the data calculated from the linear model for the soil temperature at the 1-inch depth from the air temperature, the resulting series for soil temperature is used as the uppermost boundary condition in the solution of equation (7), the heat flow equation. Van Wijk (1963) presents the theory for solving for the constants TA and TO (p. 133-138 of his book), using the Fourier series of the soil temperature at the 1-inch depth. The constant $\overline{T}$ is found by summing all the temperatures at the different depths of soil temperature for the 4 years of record used in generating the model and by finding the average value. The value of ω is 360° (2π radians) divided by the number of days in a year (365).

The damping depth (D) is found from the soil temperature measurements at the different depths for the 4 years of record in the following manner. For a given depth, the difference between the maximum and minimum temperatures on a monthly basis (monthly averages were computed) for the whole year at each measured soil depth were found for each of the 4 years of records. The natural logarithms of the difference between the maximum and minimum monthly values at each depth were taken. If a plot of the natural logarithms vs. soil depth is made, the slope of the line generated is equal to -1.0/D. A least
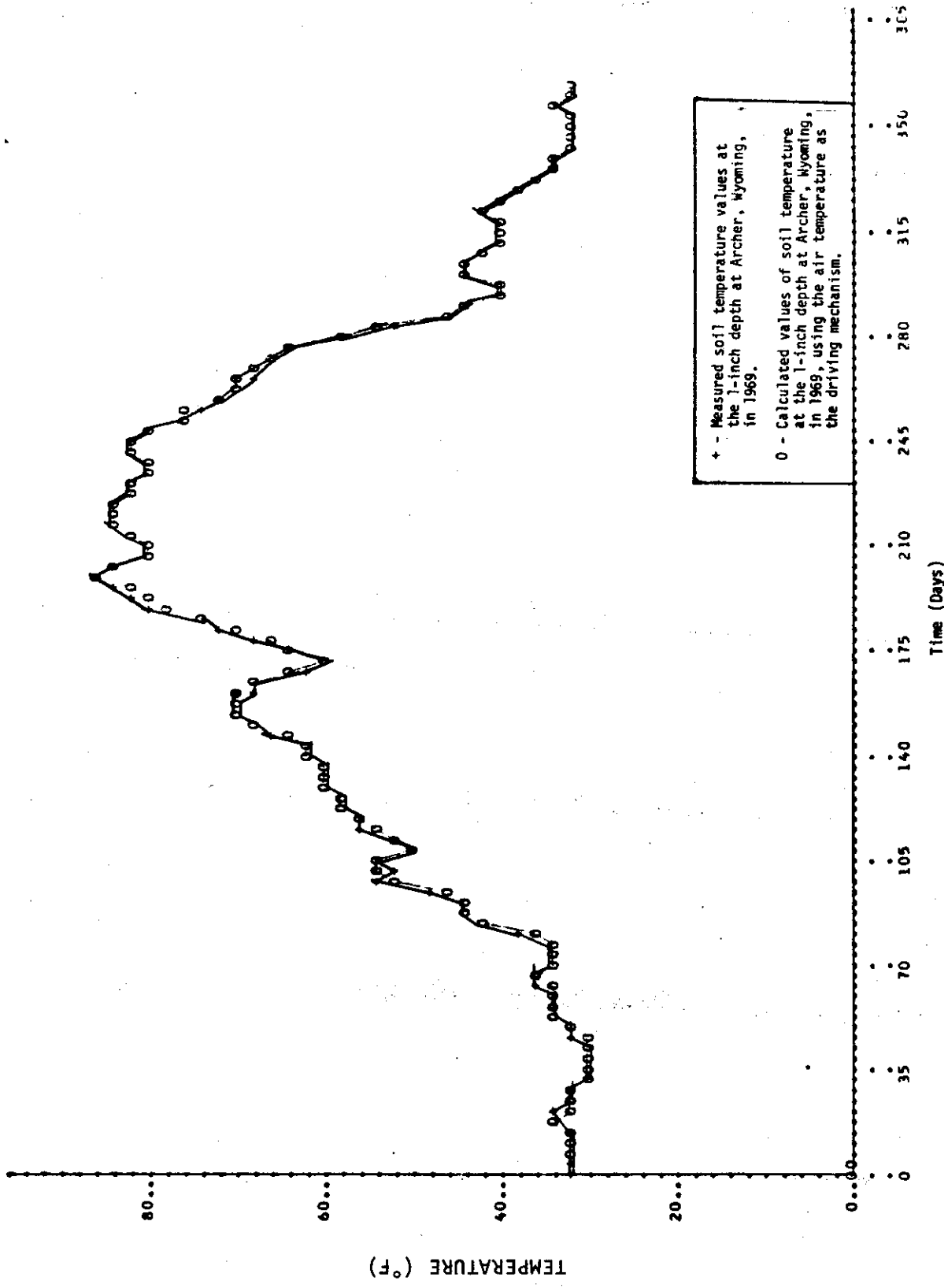
Fig. 4. Comparison between measured and calculated soil temperatures at the 1-inch depth.

squares approach was used to determine the slope of the line. The damping depth (D) is then found by taking the negative reciprocal of the slope.

The constants in equation (7) have all been determined using the above procedure. Equation (7) can now be used to determine the soil temperature at any depth for the Archer Site. Soil temperatures at depths of 2.25 inches, 4.0 inches, and 8.0 inches were determined from equation (7) for the 3 years of record not used in generating the constants of equation (7). Fig. 5, 6, and 7 show the predicted or calculated values of soil temperature at 2.25 inches, 4.0 inches, and 8.0 inches, respectively, for the Archer climatolog- ical station in 1969. Values of the measured soil temperatures at each of the above depths is also shown on their respective figures for the soil depths after a 10-day moving straight average was applied to the measured data.

Using the soil temperature model generated in the preceding pages for the Archer, Wyoming, climatological site, an extensive analysis using this model is now planned for the Pawnee Site. With this model and the data being obtained from the Pawnee data acquisition system, a complete soil temperature model for the prediction of soil temperatures on a daily, hourly, and even minutely basis will be constructed for the Pawnee Site. A model similar to that presented by Hanks et al. (1971) will be used to describe the diurnal cycle, using the model presented in this report to determine the mean daily soil temperature at the 1-inch depth.

All calculations were made using the Sigma 7 computer at the University of Wyoming Computer Center.
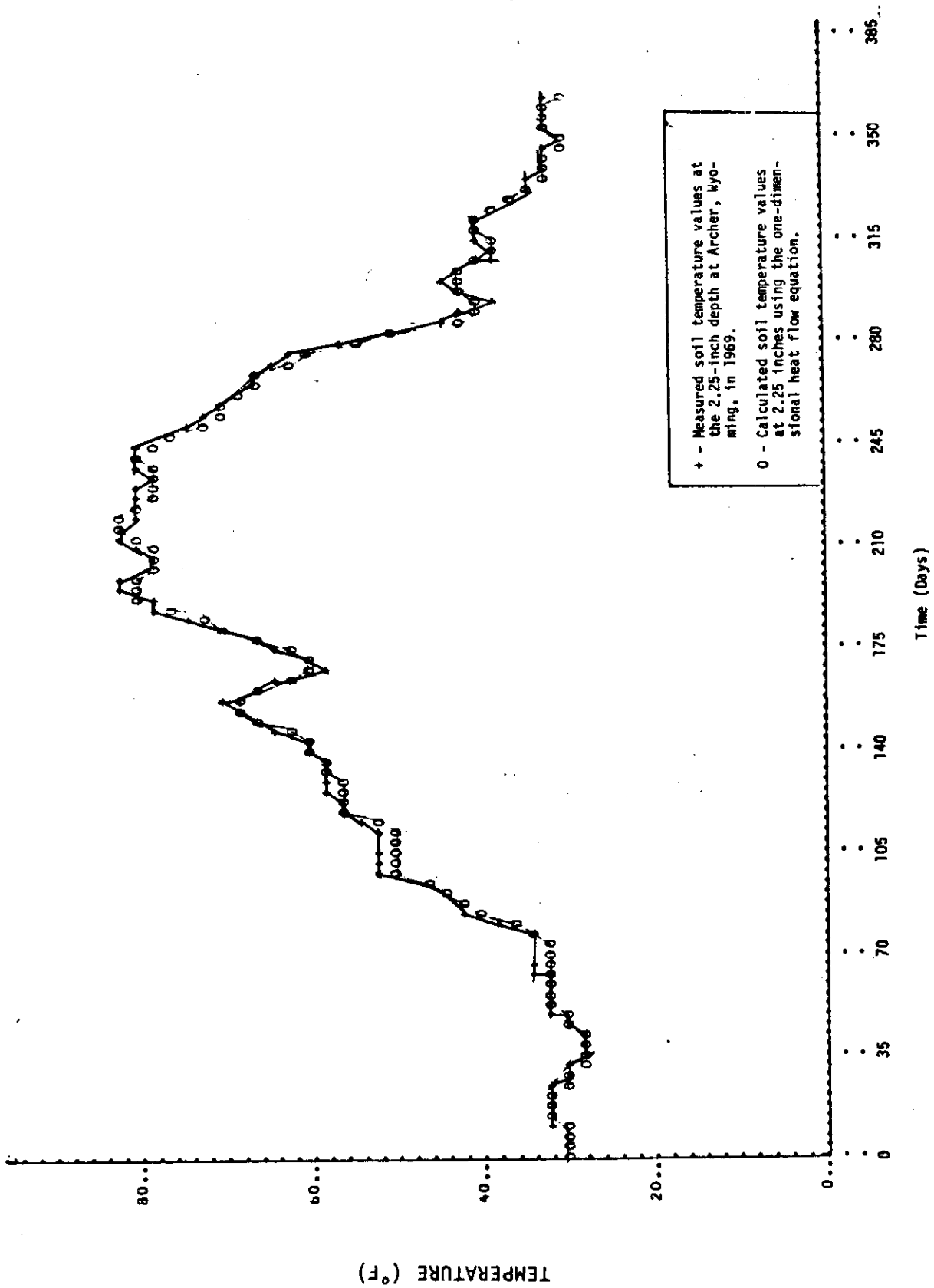
Fig. 5. Comparison between measured and calculated soil temperature values at the 2.25-inch depth.
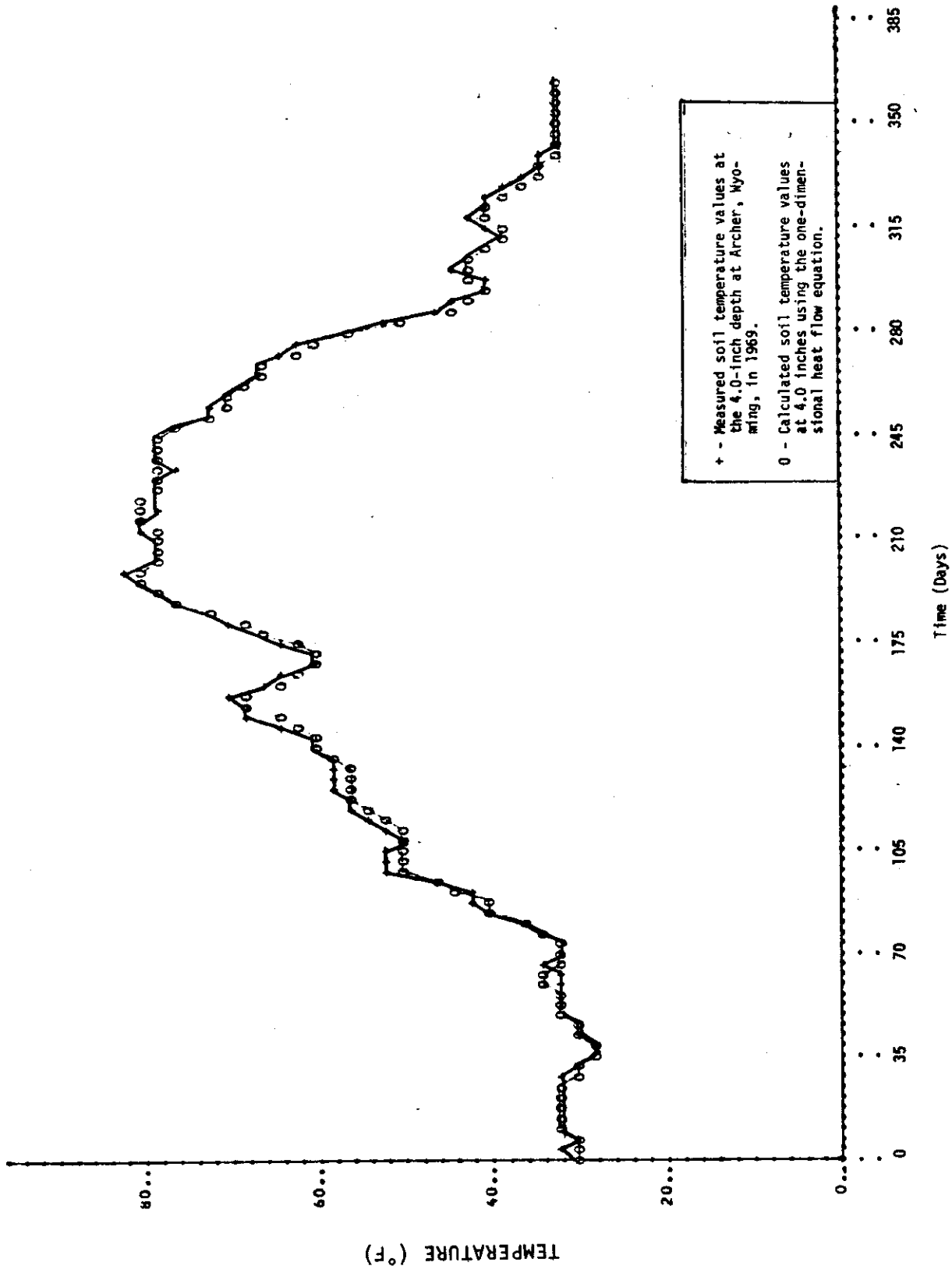
Fig. 6. Comparison between measured and calculated soil temperature values at the 4.0-inch depth.
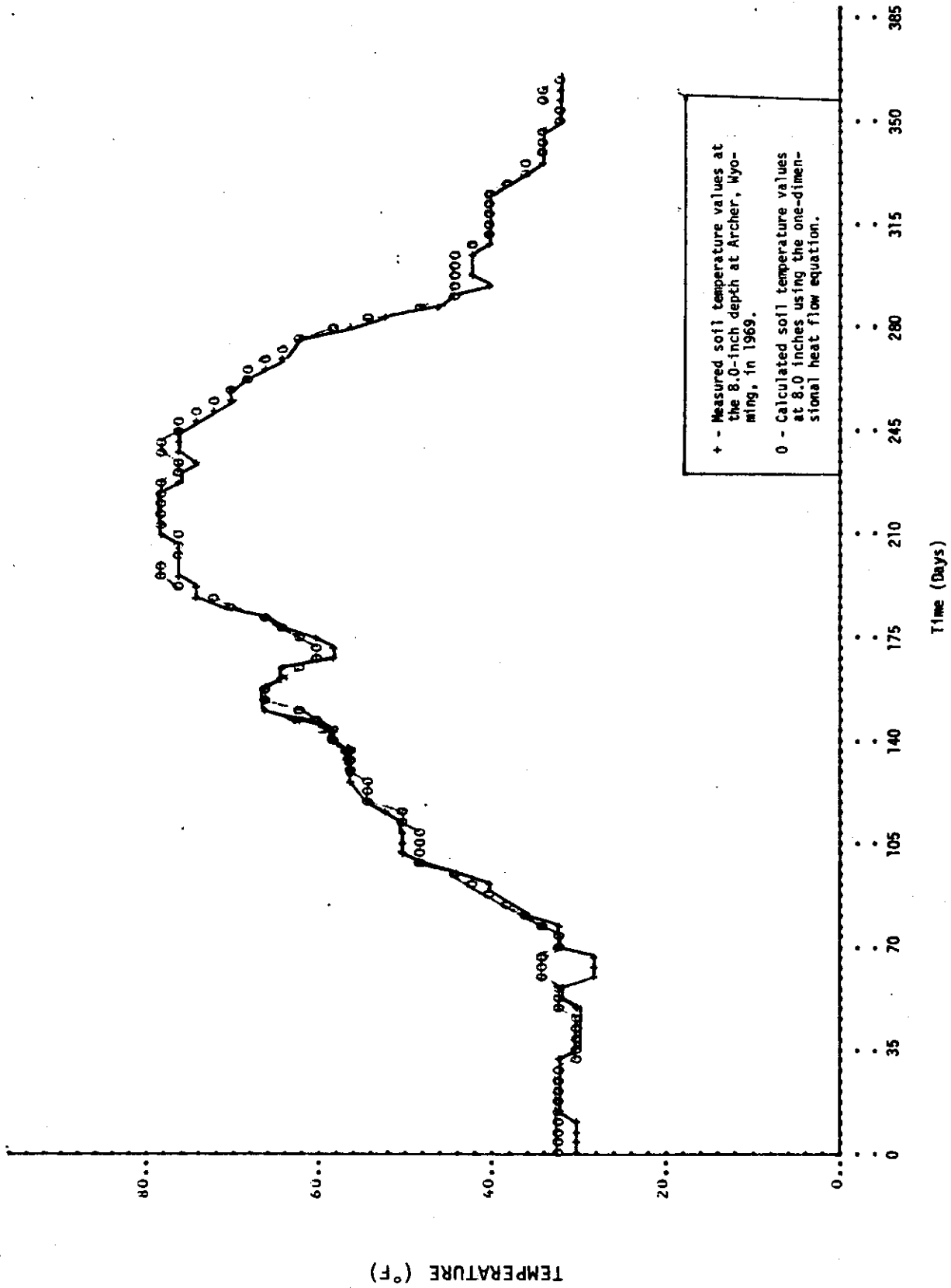
Fig. 7. Comparison between measured and calculated soil temperature values at the 8.0-inch depth.

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