DISSERTATION

ESSAYS ON FINANCING REFORM AND THE PROVISION OF LOCAL PUBLIC EDUCATION

Submitted by

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ABSTRACT

ESSAYS ON FINANCING REFORM AND THE PROVISION OF LOCAL PUBLIC EDUCATION

This dissertation investigates the provision of local public education from both theoretical and empirical aspects. In the theoretical sections, the existence of Pareto-improving reform (to redistribute education resources away from the rich community and toward the poor community) is examined under the current public education financing system. In the empirical section, the state financing system on public education in the state of Colorado is tested.

Chapter 1 introduces the importance and motivation of my research topic.

Chapter 2 directly follows the theoretical framework in Fernandez and Rogerson (1996). As far as I know, they are the first to examine the provision of public education under a multi-community and multi-income-group model and discuss reforms which might be Pareto-improving. By adding additional assumptions on population distribution and individuals' preferences, I analytically show that under a two-community and three-income-group model, when local public education is financed by an income tax, the reform "to redistribute a fraction of education expenditures away from the rich community toward the poor community" is Pareto-improving.

Since public education is mainly financed by a property tax, a general housing market with an upward sloping supply curve is introduced in Chapter 3. Simulations show that when local public education is funded by a housing property tax, the reform posed in Chapter 2 may still work. The redistributive fraction chosen by the state government determines whether the reform is Pareto-improving or not.

In the empirical section, Chapter 4, I develop four regression models to examine the effects of the state financing policy on public education in Colorado. The results show that the Colorado state government is reducing disparity in per student spending across school districts. However,

the current policy is not potentially Pareto-improving according to the theory developed in Chapter 2. Thus, policy suggestions are made.

Chapter 5 summarizes and concludes my dissertation.

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While all contributions helped enormously in the process of this dissertation, any errors are my own.

DEDICATION

I dedicate this dissertation to my parents for nursing me with affections, your love is the most valuable treasure in my life.

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CHAPTER 1

INTRODUCTION

Today among all K-12 education methods (private, public and homeschooling) in the U.S., public education is dominant. According to data from *2013 National Center for Education Statistics*, almost 90% of school-aged children were sent to public schools in the year 2005, and the percentage seems to continue growing. Without any exaggeration, the quality of public education plays an important role in determining the future of our younger generations.

One noticeable fact about the public education is the great disparity in per student spending across school districts. Evidence from *National Center for Education Statistics* and *Colorado Department of Education* show that, even in the same area (county), the average education expenditures in a rich school district can be more than twice as much as that in a poor school district. This might be one of the most striking features of the public education system in the U.S. today.

Another fact about the public education in the U.S. is that in most states, public schools are mainly financed under a "mixed financing system," where state and local tax support counted more than 90% of the total funding and the share of federal support is less than 10%[1].

During past decades, debates have risen about the great disparity in education expenditures across districts. Educationists and sociologists care more about the equality of the public education while economists focused on the efficiency[2] of the financing system. The tradeoff between equality and efficiency has always been one of the hottest topics in the debate, school financing equalization will significantly reduce per student spending in rich districts and greatly hurt the efficiency of the public education in those districts. However, if we keep the education system at

[1] Source: U.S. National Center of Education Statistics, 2014.

[2] In many related literature (Fernandez and Rogerson 1999 and Downes and Schoeman 1997), households' (total) utility or individuals' (total) utility is used as the measurement of the efficiency. In my dissertation, I am also going to use utility function to measure the efficiency of the financing system.

an efficient level, we can hardly narrow the gap of average education expenditures between rich districts and poor districts.

McClure et al. (2008) argued that the state government should do more to reduce the inequality in public education (e.g., the state government can increase the state funding for poor districts and decrease the funding for rich districts). Berg et al. (2011) are worried that children from poor households who cannot attend good public schools may be stuck in a poverty trap under the current education system.

Fernandez and Rogerson (1999) argued that further reform may hurt the efficiency of the public education financing system by greatly decreasing the education expenditures in rich districts. Downes and Schoeman (1997) have found evidence that under further reform, the whole public education sector might be hurt. This is because the rich households will send their children to private schools when they have noticed a significant fall in public school expenditures in their own districts, which will cause education resources (e.g., teachers, funds and policies) to flow from public schools into private schools.

In my dissertation, I try to strike a balance between the equality and efficiency debate about the financing of public education. The major goal of my theoretical research is to provide insights into the effects of the current financing system for public education and to examine the existence of (potentially) Pareto-improving reforms. In the empirical section, I analyze the state education financing system in Colorado and make policy suggestions for the state government whose goal is potential Pareto-improvement.

Chapter 2 of my dissertation directly follows Fernandez and Rogerson (1996). The authors built a model with multi-community and multi-income-group features to examine the provision of local public education and discussed potentially Pareto-improving reforms. In order to analytically solve the Pareto-improving question posed by Fernandez and Rogerson (which was not provided in the original paper), I add two assumptions on population distribution and individuals' preferences. By using individual's total indirect utility as the Pareto criterion, I show that in a two-community and three-income-group model, the reform "to redistribute a fraction of education expenditures away from the rich community toward the poor community" is Pareto-improving. Fernandez and Rogerson used local income tax to finance public education in their original model. In my second chapter, I also examine the case when public education is funded by local property tax revenue. My analysis shows that the reform is still Pareto-improving under a perfectly competitive housing market. However, with given initial income, the property tax provides less average education expenditures than the income tax. At the end of this chapter, I derive the necessary condition of Pareto-improvement when there are more than two communities in the economy: the migration pattern of individuals moving from wealthy communities to less wealthy ones.

In Chapter 3, I introduce a general housing market (a housing market with an upward sloping supply curve) into the model and use the housing property tax revenue to finance the local public education. Two-community model is still used in this chapter, while the initial incomes for the three income groups are generated based on income census from American Community Survey (ACS). Since the model is too complicated to solve analytically, numerical methods are employed to analyze the effects of the redistributive policy posed in Chapter 2. The benchmark simulations partly follow Fernandez and Rogerson (1998), and results show that the same reform is still Pareto-improving if the redistributive fraction chosen by the state government lies in a certain range. I also test the impacts of the reform under different migration rates, the results can be used to examine the welfare change of major counties in Colorado.

In the empirical chapter, I conduct a study of the state education financing policy in Colorado. Panel regressions are used to examine whether the Colorado state government is redistributing education resources away from the rich school districts toward the poor school districts and whether the current policy is potentially Pareto-improving. My regression results show that the state government is reducing disparity in per student spending across districts, however, the current policy does not satisfy the necessary condition of Pareto-improvement. Under current policy, when local wealth increases, the increase in local education support is greater than the fall in state education support, and the total effect on local per student spending is positive. Thus, households will find it beneficial to move to wealthier districts, which is not potentially Pareto-improving according to the theory in Chapter 2.

The major contribution of my theoretical chapters is uncovering the evidence of Pareto-improvement under the current financing system on public education. It provides confidence to people that we can help the poor districts by increasing average education expenditures while not hurting individuals/households in the rich districts. The results in the empirical chapter can be used as a guidance for policy makers when the goal of state education financing system is to make a potential Pareto-improvement.

CHAPTER 2

EDUCATION FINANCING REFORM AND PARETO-IMPROVEMENT UNDER DIFFERENT TAXES

2.1 Introduction

Public education has always been one of the hottest policy subjects in various occasions. In Congress senators from different political parties have fierce debates on whether the federal government should get out of the way on local public education affairs. In February 2013, Senator Lamar Alexander, Republican of Tennessee, forcefully urged that the states should be allowed to set their own public school policies as Congress contemplates rewriting *No Child Left Behind* (Motoko Rich, 2013).

In States, governors try hard to improve the education system in order to provide the children with a better learning and growth environment. In August 1971, the California Supreme Court ruled in the case of Serrano v. Priest that the old finance system which based education finance on local property wealth and therefore is considered to discriminate against the poor and reforms need to be put forward (Durbin, 1972).

In many research institutions and universities, researchers, scholars and experts in sociology, pedagogy, ethics and economics make their own suggestions on public education. And in some households, parents would have a disagreement on which school should they send their seven-old-boy to.

Although in the U.S. there are other choices for a child's education such as private school and homeschooling, most school-age children are sent to public schools. According to the survey of *National Center for Education Statistics*, in the fall of 2015, the expected number of students will attend the primary and secondary public schools is about 51 million, among those students, 36 million will be under 8th grade and the remaining 15 million will be in "upper secondary schools," or quite literally, high schools, while the number of students expected to attend PK-12 private schools is only 5.2 million.

As can be seen from Table 2.1, since the year 2005, nearly 90% school-age children are in public schools and the percentage is still growing, which is largely due to the fact that the number of students accepting private education has been shrinking.

Table2.1: PK-12 students' enrollment in public and private education systems

Year[a]	Total	Public			Private[b]
	PK-12	PK-12	PK-8	9-12	PK-12
2005	55,187	49,113	34,204	14,909	6,073
2006	55,307	49,316	34,235	15,081	5,991
2007	55,203	49,293	34,205	15,087	5,910
2008	54,973	49,266	34,286	14,980	5,707
2009	54,862	49,373	34,418	14,955	5,488
2010	54,876	49,484	34,625	14,860	5,391
2011	54,956	49,636	34,849	14,787	5,320
2012	55,091	49,828	35,076	14,752	5,263
2013	55,288	50,067	35,301	14,766	5,221
2014	55,599	50,407	35,502	14,905	5,192
2015	55,957	50,773	35,735	15,038	5,183

Source: 2013 National Center for Education Statistics

[a] All of these data are from the website of National Center for Education Statistics. The data before year 2011 are actual numbers and from 2011 to 2015 projected numbers are used.

[b] Includes private nursery and prekindergarten enrollment in schools that offer kindergarten or higher grades.

NOTE: PK=prekindergarten, students numbers measured in thousands.

Given the truth that most of our children will be taught in public schools, it is not reckless to say that primary and secondary public education is the foundation of national education and determines one nation's future to a certain extent. The education system in the U.S. is divided into three stages: primary education, secondary education and tertiary education (U.S. Department of Education). K-12 education is the prerequisite of higher education as it provides students with the environment for healthy physical and mental development as well as the basic ability to learn systematically (Blundell, Dearden, and Sianesi, 2005), which is essential in both university studies and job markets. K-12 education also helps build the cornerstone of the whole society, it instills acceptable ethics, values and laws in our children, and thus they will be good citizens when they grow up (Elkind, 1976). What is more important, equipped with knowledge and skills, many of these future adults will become good workers in industries and elites in research fields, making a contribution to the development of human civilization.

There has been a growing disparity in primary and secondary public education across the country. This phenomenon is illustrated in Table 2.2. The numbers in the table show great variations on expenditure per student not only across different states but also among different school districts within the same state. Spending per student on elementary and secondary education in one school district can be twice as much as the expenditures per pupil in another school district, which is one the most striking features of the public education system in the United States.

It is not surprising the gap of expenditures per student between different school districts could vary so greatly. At the national level, typically more than 40%[3] of the total education expenditures for each school district are from local tax revenue, and most the remaining support comes from state governments. Public education in most school districts in the U.S. is funded under this mixed financing system[4].

[3] Source: U.S. National Center for Education Statistics, year 2000 to 2009.

[4] Although school districts do receive certain amount of education tax revenues from federal government, the percentage is tiny (less than 10% according to data in National Center for Education Statistics). In this paper when "mixed financing system" is used, it refers to a mixture of local and state financing.

Given that public education is mainly financed at the local level, the disparity in average expenditures across districts actually reflects the disparity in wealth among different income groups across different school districts (communities). Although the mixed financing system does reduce the disparity in education spending to a certain extent (the state government financing works as redistribution), it receives criticism from both sides.

Table 2.2: Example of total current spending per pupil[a]

Jefferson, AL[b]		Apache, AZ		Alameda, CA	
Concho	13,593	Homewood	1,1855	Emery Uni	15,604
Vernon	12,753	Birmingham	1,0676	Berkeley	12,429
St. Johns	9,813	Tarrant	9,587	Hoover	10,452
Round Valley	8,233	Midfield	9,028	Albany	9,168
Mcnary	7,148	Jefferson	8,872	Hayward	8,646
Chinle	7,074	Trussville	8,635	Newark	8,101
Sanders	6,925	Fairfield	8,545	Fremont	7,541

Source: 2013 Census of Government
[a] These are elementary and secondary data for the academic year 2010-2011.
[b] The first row is the counties' name, column 1, 3 and 5 are school districts with each county.

The advocates of further reform do not think that governments have done good enough (McClure, Wiener, Roza, and Hill 2008) to reduce the disparity in public education spending across different districts. They argue that under current financing system, the households from the bottom of income groups with a poor quality of public education available will be stuck in a poverty trap (Berg et al 2011). These income groups cannot afford the cost of living in an area with a better school district. The money spent per student in a good school district is much more than those spent per student in a poor school district. This allows the schools in rich areas to afford better facilities and teachers, which count as two of the most important components of the quality of public education. Students receive a higher quality of K-12 education are more likely to enter better universities and thus are more likely to find well paid jobs meanwhile many young people from poor districts who could not even get a bachelor's degree will continue struggling in the bottom of the society as their parents did in the past (Berg et al 2011).

This potential reality is condemned fiercely by many people not only because it goes against the idea of equal opportunity in a market economy but it also may violate the Fourteenth Amendment to the U.S. Constitution.[5] Although different people are endowed with different wealth, talents and even different fortunes, the government should create the opportunities as fair as possible for as many people as possible such that those who working hard can always find their own ways to achieve personal successes.

The voice from the other side also needs to be paid attention to. Some economists are worried that if the state governments take the major responsibility in the financing system, more resources for public education in the rich communities will be redistributed toward the poor communities, and the efficiency of the financing system will be hurt (Fernandez and Rogerson, 1999). As the households live in rich communities are much less than those live in poor communities, the average increase in expenditures per student due to the redistribution in poor communities is far from a desirable level meanwhile the spending per student in rich communities falls significantly. What is worse, further reform may hurt the public education sector as a whole (Downes and Schoeman, 1997). When rich households see the quality of public education decreases, they will send their children to private schools. This further causes the resources for education flow from public sector to private sector (e.g., teachers, funding, and relative policies).

For those households with school-aged children, the choice of where to reside is influenced by the quality of the school district. Although there are exceptions, very few families[6] choose to

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[5] According to the Fourteenth Amendment to the U.S. Constitution, education is a fundamental right and every one should have equal access to it.

[6] According to the latest data from *National Center for Education Statistics*, in year 2007, among students who received public elementary and secondary education (88.7% of school-age children chose to attend public schools), 82.5% of them went to assigned schools.

send their children to a different school district because doing so means they need to pay extra money for transportation and to wait on a waiting list until the schools they choose have enough teaching resources (classroom space or teacher). The quality of public education one community can provide is closely related to the wealth conditions of its residents. The richer communities provide their residents with a better living environment as well as better schools, all of which are built on higher tax revenues and the rich households who can afford them. On the other hand, families who cannot afford high taxes and expensive properties have to live in communities with lower tax rates, lower housing prices, fewer tax revenues as well as a smaller average tax base. Thus the quality of public education provided in poorer communities is less than that of the wealthier communities (Berg et al 2011).

So when the provision of public education is to be examined in the economy, it is of great importance to take multiple communities into consideration not only because public education can hardly be provided at the same level in different communities but also because different communities are characterized by their own mean income, tax rates, population which further have impacts on the provision of public education.

Communities compete with each other in attracting households and people are always choosing to live in the districts with the best combination of "costs and benefits" for them (Tiebout, 1956). The main costs for most families are taxes they need to pay and the expenditures on housing in order to live in one community. One of the major benefits for households with children is the quality of public education provided by local schools. When the combination of "costs and benefits" has changed in certain communities, some households find that their community becomes less attractive compared to others. This may cause households to change communities and migration will happen.

Among all the income groups, households from the top and bottom are not very likely to move. According to the data of *Census of Governments*, from the year 2011 to the year 2012, for all more than 15-year-old householders, the number of non-movers decreases as households' income increases from less than $5,000 to $60,000 and then increases rapidly. For those low-income families, migration cost and the cost of living in a better community are too high for them that prevent them from moving to other communities (Dacanzo, 1976). On the other hand, although high-income families can easily afford the migration cost, their population is small and only a few number of communities with best amenities are attractive to them. Middle-income household groups, who constitute the biggest part of the population and reside in most of the communities, are sensitive to those changes since the number of communities available to them is large and families in those communities have relatively close preferences (Benabou, 1996). In addition, migration may have a snowball effect, once some households migrate from one community to another, characteristics such total tax revenue, the population in both communities also change, resulting in migration of other households. Thus, if there's no or little migration cost (we could consider the case as households moving between adjacent communities), new education policy implemented in one school district, all else equal, may lead to large population migration in that area, which is very likely to have further impacts in the initial districts. So mobility of the households plays a very important role in determining the final effect of any local policy change on the community and the area nearby.

In most states, the pace of the education reform is slow. During the 1970's California took the first step towards reforming the public education financing system because the district-to-district disparity in funding public education "fails to meet the requirements of the equal protection clause of the Fourteenth Amendment to the United States Constitution and the California Constitution" (Serrano I, 1971). The state government switched from the mixed financing system to the purely state financing system. Although the distribution of education expenditures became more equal, from economists' point of view (Rogerson, 1999 and Schoeman 1997), this reform is not completely successful because of the efficiency loss on funding the public education. So the reality puts the reform on public education into a dilemma: the current mixed financing system has been criticized a lot due to the existing disparity in education spending while the reform took place in California did not have desirable outcomes from economists' point of view.

Thus one question to be asked is that can we strike a balance between equality and efficiency in the provision of public education under current financing system?

2.2 Research Plan

The goal of this chapter is to provide a model in which a redistributive policy aimed at reducing education inequality is Pareto-improving. My theoretical framework directly follows Fernandez and Rogerson (1996). They built a model simple enough yet captures all the factors that are concerned: different school districts (communities), different income groups, individual and household preferences, migration, local public education and governments.

In Fernandez and Rogerson's multi-community and multi-income-group model, individuals choose which community to live based on their initial income. Each community is characterized by an endogenous proportional income tax rate (determined via majority voting) and by the corresponding quality of public education. The quality of public education in one community is determined by the education expenditures per student in that community.

Fernandez and Rogerson generally discussed various of policy reforms and potential Pareto-improvement. In order to make the Pareto-improving analysis mathematically solvable, I focus on the two-community and three-income-group case. Additional assumptions on population distribution and individuals' preferences are made to exclude tax change caused by migration because tax change will make the utility change intractable in a model without explicit functional forms. Individuals' indirect utility is used as the Pareto criterion. In addition, I also examine the case when housing property tax is used to finance the public education.

I use the model to examine one of the redistributive policies suggested by Fernandez and Rogerson (1996): to redistribute a fraction of education expenditures away from the rich community and toward the poor community. The result of my analysis shows that if the fraction is correctly chosen, the policy, aimed at subsidizing the public education in the poor community, can be Pareto-improving. Thus, we can narrow the gap of public education between rich and poor communities and make some people better off while others maintain their well-being level. Although this one single model is not enough to guide policy making, by itself it does provide insights on solving the "efficiency-equality" dilemma and directions for future research.

The robustness analysis indicates that the impact of the redistributive policy is sensitive to the population distribution of different income groups. Since in each community the tax rate is determined via majority voting, the population ratio between two income groups within one community plays the only role in determining the tax rate. Once the migration causes the voter in the poor community to change from a low-income individual to a middle-income individual (this is one of the possibilities when we have a different population distribution), the welfare change for low-income individuals is ambiguous. This is because the middle-income voter will choose a higher tax rate, which decreases low-income individuals' first-period consumption. However, my analysis also shows that the local government can use regulations to ensure that the redistributive policy is Pareto-improving.

This chapter is organized as the following. In section 2.4, I start with Fernandez and Rogerson's (1996) model, introduce the additional assumptions, and describe the characteristics of the "voting equilibrium" (I call it "initial equilibrium" in my paper). Section 2.5 provides the Pareto-improving analysis and how the Pareto-improvement question can be solved mathematically. In section 2.6 I extend section 2.5 by using a housing property tax. In the last two sections, a comparison between the results of two types of taxes and the sensitivity analysis is made.

2.3 Literature Review

The foundation of my theoretical framework is built on Tiebout (1956), *A Pure Theory of Local Expenditures*. Tiebout made assumptions about how local governments provide public goods in a world with multiple communities. These assumptions are widely used in the multi-community analysis after Tiebout, and some of them also capture the key features of my model. ①There is no migration cost so economic agents can freely move to the community where their utility can be maximized. ②The economic agents have full knowledge of all consequences of any policy change and will response to them quickly. ③There is no unemployment in the economy and all economic agents are endowed with a certain amount of initial income. ④Economic agents make their location decision based on a large number of communities. ⑤The locally provided public goods have neither positive externalities or negative externalities between two communities.

Westhoff (1977) mainly criticized two of Tiebout's assumptions. First, there are enough communities for a complete spectrum of different public good and tax combinations. Based on this assumption, we might have the case that some communities are constituted by homogeneous consumers, more importantly, an arbitrary number of communities cannot guarantee the existence of equilibrium. The second critique focused on the *U-shaped* function of the per capita cost of a local public good. Since public goods are non-exclusive, the *U-shaped* function makes the good more like a private good. Due to the scarcity of community size and the fact that the number of communities for individuals/households is always limited, Westhoff assumed the consumers have to choose to live among finite communities. The direct result of this assumption is that as some communities are constituted with heterogeneous consumers, and therefore disagreement over the provision of public good and the tax rate will occur. Westhoff employed the method of majority voting to obtain the unique tax rate (this method is used in many multi-community models with heterogeneous economic agents when the tax rate needs to be determined within one community). In addition, it is also assumed the public good is a Samuelson pure public good. Westhoff's contribution showed if restrictions are placed upon consumers' preferences (continuity and convexity), the voting equilibrium, which is defined as "no consumer is better off by migrating to another community," exists in the multi-community model with the provision of a public good.

Epple, Filimon and Romer (1984) introduced the housing market into the multi-community model and it is assumed that the utility function, which is a function of public good consumption, housing consumption, and private consumption is identical for every individual and the provision of public good is financed by a housing sales tax (unlike Westhoff who used proportional income tax to finance the public good). They found the three necessary conditions for the existence of intercommunity equilibrium. The first is s*tratification*, or that each community is formed of individuals with incomes in a single interval. In other words, we cannot have the case that low and high-income people live in one community while middle-income people live in another community. *Boundary indifference* is imposed when the boundary consumer is indifferent between the two 'adjacent' communities. Finally, *ascending bundles*, means that the community where rich individuals choose to reside tend to have both higher provision of public good and higher gross housing price than the community chosen by relatively poor individuals. Epple, Filimon, and Romer further assumed that the cost of providing the public good is a linear function of the quantity of the public good and the population of the community, and individuals will not choose the consumption bundles in which any good is not consumed. With these three necessary conditions and restrictions on the technology of public good supply and consumers' preferences, it is sufficient to show the existence of the intercommunity equilibrium. Epple, Filimon, and Romer ran a simulation to examine their theoretical results. The number of communities is simplified to two, utility functions are assumed to be Cobb-Douglas form, and some parameter values are from Mills (1972). The numerical examples demonstrate a unique, stable equilibrium exists for a variety of parameter values.

Epple and Romer (1991) examined the redistributive effect in a multi-community model in which there is no migration cost and individuals can freely move among different communities. Housing and a numeraire bundle are assumed to be the only two goods in the economy, and there is no provision of the public good. Majority voting determines the proportional tax on the value of housing within each community and the tax revenue is used to pay a lump sum to every resident in the community. The computable equilibria of the model show that the communities with the poorest households and large communities tend to have greater redistribution. This is because households with the lowest income are also the ones need the largest subsidy, communities with larger population can also collect greater tax revenues and use them for redistribution.

The early literature focused on building a multi-community model with proper assumptions on the features of the economy, consumers' preferences, and the provision of public goods so that the intercommunity equilibria exist. Some researchers also analyzed the redistributive effect within the community. These papers discussed the public goods in a very general way and have no specifications on what the public good is.

Since my model is closely related to a growing research on the financing system of public education, first I want to make it clear what factors are important if the locally provided good is public education. Evidence from empirical papers shows that personal income/wealth plays a key role in financing public education. Feldstein (1975) built a log-linear regression model and found that the expenditure per pupil in one community is positively related to the wealth per pupil in that community. Fernandez and Rogerson (1997) used a panel data set for different states from 1950 to 1970 and modeled the impact of growth in personal income on per student expenditure on public K-12 education. Their results indicated that average education expenditures tended to grow at nearly the same rate as personal income per student. The conclusion drawn from Corcoran and Evans (2010) is in contrast with many theoretical and empirical works. They suggested that income inequality that decreases the median voter's tax share results in higher expenditure on public education. The estimation shows that 12% to 22% of the growth in local public education spending from 1970 to 2000 can be attributed to increasing income inequality. The explanation for this is that median voter's tax share decreases as income inequality increases, which results in higher local public education expenditures.

Glomm and Ravikumar (1992) developed an overlapping generations model in which human capital investment through formal schooling is the source of growth. Two education systems are examined for the accumulation of human capital. Under the public education system, every individual faces the same quality (measured by expenditures per student) of education. This is exactly the same with the public education financed at the state level when the state government provides the same education expenditures per student across the state. Private education system is very similar to the system when public education is financed mainly at the local level because in both cases rich individuals will benefit from a high quality of education. The analysis shows income inequality declines more quickly when education is uniformly provided and private education results in higher per capita income.

Inman (1978) used data from New York metropolitan area and ran simulations based on a multi-community model to examine seven reforms on public education. In the first, *Foundation Aid (FA)*, the state government subsidizes each school district a lump sum amount per child. In *Foundation Aid with a Spending Limit (FALIM)*, on the other hand, each school district receives a foundation level of subsidy per child. Under *District Power Equalizing Aid (DPE)*, a target fiscal base per child from which it can fund its education expenditures is set for each district. Under *DPE Aid with No Recapture (DPENC)*, which is identical with *DPE* except that the aid level cannot be negative. *Match Aid (MA)*, within one state, all districts share the costs of local school spending at a certain rate. Under *Tax Credit (TC)*, a property tax credit is given to each family against the state income tax they need to pay. Finally, under *Financing Fiscal Reform*, a proportional state income tax is used to pay for various transfers. The simulation results show that different reforms are preferred under different social welfare criteria. *DPE* is the best under a Benthamite utilitarian criterion. *Foundation Aid* should be chosen under the Rawlsian rule. Finally, *FALIM* is ranked number one under Atkinson's inequality measure.

Fernandez and Rogerson (1996, 1997 and 1998), to the best of my knowledge, are the first to use a multi-community model with the provision of public education to analytically evaluate different education reforms. Fernandez and Rogerson [1996] first proved the existence of equilibria and listed the characteristics of the equilibria based on the assumptions they made. In the stable equilibria of their model, people stratify themselves into communities according to their initial income. No community is empty and each community is characterized by an endogenous proportional income tax rate and the corresponding quality of public education. Richer communities are those communities with higher mean income, higher tax rate, and higher education expenditures per student. To analyze the impact of different financial reforms on public education, comparative statics exercises are employed and the model is simplified to a three-income-group and two-community case. Their analysis shows that policies whose net effect is to increase the number of wealthier residents in a relatively poorer community will tend to be Pareto-improving. As mean income in each community increases, so does the quality of public education and the tax rate will decrease. The robustness of this result is examined. It indicates that many of the reforms will also work in a more generalized multi-community and multiple income-group case.

However, it is difficult to prove those potential Pareto-improving reforms suggested by the authors are actually Pareto-improving because of the following. Firstly, Fernandez and Rogerson used both tax rate and quality of public education as the Pareto criteria. In analyzing the impacts of the reforms, it is difficult to tract two things at the same time. More importantly, since tax rates for each community are determined by majority voting, they are very likely to change when policy reforms cause migration, the direct result of which is that the utility change for certain income group is ambiguous. Take the two-community and three-income-group case for example. When the redistributive policy causes middle-income individuals moving from the rich community to the poor community, the mean income in the poor community increases while the median voter in the poor community may change from low-income individual to middle-income individual and a higher tax rate will be chosen. So for the low-income individuals, their current consumption will decrease due to a higher tax rate but future consumption will increase because of a higher average spending on public education caused by higher mean income. If this is the case, it is very difficult to tell whether the utility for a representative member of the low-income group increases or decreases.

Fernandez and Rogerson (1997) examined the zoning effect. They added zoning into the model to capture the fact that low-income families cannot afford the living cost in a rich community. Housing and private good markets are introduced into the model, so there are three components of one individual’s utility: private consumption, public consumption (public education) and housing consumption. The tax revenue used to finance the public education is collected by imposing a housing sales tax rate, the supply side of the housing market is exogenous and perfectly competitive, and individuals' initial incomes no longer follow a discrete distribution but are characterized by a continuous density function. A two-community model is presented, and simulations are used to examine both exogenous and endogenous zoning effects: individuals are required to purchase at least some amount of housing in order to live in the rich community. In the exogenous case, this amount is chosen by the third party while in the endogenous case the individuals in the rich community choose the amount to maximize their utility. The analysis shows that for both cases, the rich community becomes more exclusive, the change of the total social welfare is ambiguous because some individuals are better off while others are worse off and how the quality of public education and tax rate change does not follow the same pattern in two cases.

Fernandez and Rogerson (1998) also examined the reform of switching the financing system for public education from purely local financing to purely state-level financing. An intergenerational dynamic model was built and they used quantitative methods to evaluate education finance reform. In the benchmark model, the individuals are assumed to live for two periods, the households are the economic agents and each consists of one parent and one child, thus there is no population change. The number of communities is restricted to two and household's utility is based on of private consumption, housing consumption and public education received. Public education is financed by a housing property tax, households are renters, and housing will depreciate completely at the end of every renter's lifetime.

To run the simulation, a transformation of constant elasticity of substitution utility function is used for the specifications of utility function and income function. Data for housing and education spending shares are taken from the *Economic Report of the President* and *Statistical Abstract of the United States*. Elasticities of housing demand, education expenditures with respect to mean income and mean earnings with respect to education quality (which is denoted as the per student spending) are from previous literature such as M. Quigley (1979), Inman (1979), Krueger (1992) and Wachtel (1976). The simulation based on the benchmark model provides a perfect stable equilibrium: both quality of the public education and gross housing price are higher in the community with higher mean income. When public education is uniformly provided by the state government, not surprisingly the per student expenditure in the second system lies between the corresponding values for the two communities in the first system, and so do tax rates and gross housing price. The unexpected result is that the total social welfare (measured by total expected utility) under state financing system is greater than that of local financing system.

However, the case study of California shows switching to purely state financing system is not totally good. Fernandez and Rogerson (1999) ran the simulation based the state data and their results suggested that although the equality of public education had been improved under the new system, large reduction in education expenditures in the richer communities only resulted in a very small increase in spending in the poorer communities. In addition, the new system caused a fall of 10% to 15% on public education funding in California, compared with the rest of the United States.

Downs and Schoeman's (1997) empirical analysis indicated that a substantial growth in the private education share in California during that time can be explained by the changes caused by that reform. They pointed out as the rich families see the expenditures per student on public education decreases and quality of schools falls, they will send their children to private schools. The potential result of this is that resources for education may flow from public sector to private sector, causing the failure of the public education system as a whole.

Fernandez and Rogerson (2003) examined other public education financing systems. Among the five systems: local, state, foundation, power equalizing with recapture (*PER*), power equalizing without recapture (*PEN*) (*PER* and *PEN* are the same systems as *DPE* and *DPENC* in Inman (1978)), the quantitative results showed that *PER* is the best if all systems are ranking based on expected utility. To run the simulation, individuals' preferences are assumed to combine the restrictions of homotheticity and separability. Parameter values are chosen from previous literature such as Fernandez and Rogerson (1999) and Bergstrom et al. (1982).

Calabrese et al. (2006) developed empirical strategies to investigate the provision of local public good with household mobility under multi-community model. Unlike other literature in which majority voting is used to determine the tax rate, this paper derived the equilibrium by employing myopic voting. Data from 1980 Boston Metropolitan are used to estimate the parameters. Predictions are also made with the estimated model, generally speaking, the results fit the real data well except the predictions on tax rates. The model predicts that the property tax rates are higher in high-income communities than in low-income communities. However, observation from data shows that high-income communities tend to have lower property tax rate. The authors did not explain why this prediction is in contrast with reality, but the reason might be that the tax rate is determined by myopic voting.

Li and Zhang (2015) investigated the pay-as-you-use principle of public finance and examined whether the education subsidization is efficient (the utility of a dynastic family is maximized under the social planner). Under the pay-as-you-use principle, the government subsidizes public education by borrowing money from the future and repays the debt using future taxes. An overlapping generations model was built, and fertility, leisure and capital accumulation (both physical and human capital) were endogenized. The social optimal question results in that the pay-as-you-use principle is preferred by the planner. A numerical example is also provided to compare the pay-as-you-use principle with laissez-faire: the optimal policy generates higher education spending, growth rate, and welfare level.

In a multi-community and multi-household-group model with public education as locally provided good. Epple and Romano (2015) investigated the efficiency under three different policies: ① *Decentralized Tiebout Sorting*, ② *Centralization*, and ③ *Expenditure Equalization*. In *Decentralized Tiebout Sorting*, each community is characterized by its own housing market and public education. Under both *Centralization* and *Expenditure Equalization*, all communities have the same education expenditures per household and a property tax rate determined by majority rule. However, in *Centralization*, peer quality is also equalized, while in *Expenditure Equalization* households are sorted into different communities based on peer effects. Epple and Romano found that the latter two policies are welfare improving compare with the first one and the welfare gains from them are similar. A numerical example found that the total welfare gain is 5.9% of average household income under *Centralization* compared with *Decentralized Tiebout Sorting*.

Avery and Pathak (2015) examined the distributional consequences of school choice programs on public education. Under a traditional neighborhood assignment rule, communities are stratified based on endogenous factors such as community's mean income, housing price, and school qualities. Thus, it is very likely that a community with high quality of schools has a high housing price and only attracts rich households (because poor households cannot afford the living costs in such a community). Thus, it is expected that neighborhood assignment rule will not narrow the gap between rich and poor groups. Avery and Pathak provided school choice rule as an alternative policy and investigated its impacts on communities. Under school choice rule, all students are assigned to schools based on a lottery, no informational or logistical friction is assumed in the lottery so all school within a certain area should have the same quality (the quality level of a school is determined by the wealth condition of children who enroll in the school). The analysis shows that although the quality of schools is equalized under school choice rule, the rich households and the poor households cannot be satisfied simultaneously. If the area is not closed, part of the households (either rich or poor) will move to other areas.

Caetano (2015) estimated parents' marginal willingness to pay (MWTP) for an improvement in the quality of schools in each grade (from kindergarten to grade 12). The MWTP is measured based on the per year valuation of public school quality (a combination of school level and expenditure, neighborhood amenities and housing conditions). Caetano used 2000 U.S. census data, regressions showed that compared with middle school grades, elementary and high schools grades are more valued by parents. In addition, households with non-school-aged children prefer neighborhood-level amenities while those with school-aged children prefer school-level amenities. By calculating current cost of improving school quality, Caetano found it more than parents' valuation for all school grades. So he suggested to include the (positive) externalities of public education in future research.

The main contribution of my paper is that it provides a mathematical proof, suggesting that under the currently used mixed financing system, Pareto-improving reform exists. We could reduce the disparity in public education spending across communities and meanwhile not hurt the efficiency of the funding of public education. I will provide a multi-community model in which local public education is financed under a mixed financing system and show that with correct redistributive policy, a certain amount of tax revenue from the rich community can be used to subsidize the public education in the poor community while the expenditure per student in the rich community remains unchanged. The intuition behind is that such redistributive policy makes the poor community more attractive for middle-income individuals in the rich community and they will migrate to the poor community. When the magnitude of this outflow is sufficient enough, the increase in mean income in the rich community can reverse the fall in per student spending.

2.4 The Model

In this section, first I will introduce some features of the multi-community model developed by Fernandez and Rogerson (1996) which is used in my own model. Then I am going to add additional assumptions to build the benchmark model used to solve the Pareto-improvement question.

The essential factors in Fernandez and Rogerson's (1996) model are given as the following:

- A finite number of communities.
- Individuals with certain amount of initial incomes who can freely choose the communities they wish to live.
- Public education, which is a locally provided public good, its quality is determined by expenditure per student and the quality received by the student determines their future income.

There are j income groups and i communities in the economy, with $j > i$. Each income group is constituted by homogeneous individuals, different income groups differ in their initial income y_j (assuming that $y_1 > y_2 > y_3 > \cdots$).

All individuals in the economy live for two time periods, and they have the identical preferences given as the following:

$$u(y^1) + \beta u(y^2)$$

y^1 is period one income (initial income) and y^2 is period two income. The only goods in the economy is public education and individuals gain their utility from income. The utility function u is strictly concave and twice continuously differentiable.

Given those individuals and communities, a two-stage game strategy is followed. In period one, all individuals choose a community to live and the local government will impose a proportional income tax rate determined by majority voting and use all tax revenue to fund the local public education.

The whole process in period one might be very complicated because those factors differentiate one community from the other are brought in by the residents. After the first round of "choosing which community to live," some individuals may find out moving to the other community will make them better off and they will move. In addition, their migration is very likely to cause the migration of other individuals and those individuals will also move……this process will end when the ***Initial Equilibrium*** is achieved.

DEFINITION 2.1:

The Initial Equilibrium is the situation in which no one will be strictly better off if he/she moves to the other community.

The tax rate in each community is determined by majority voting and we have:

PROPOSITION 2.1:

Given a community and a set of individuals as its residents, the majority voting results in the preferred tax rate of the individual with the median income within the community.

Since this is a static model and the migration process is not the interest of my research, I start the analysis with the initial equilibrium. Note that according to the definition above, the initial equilibrium is stable.

In period two, individuals pay the taxes, receive education and earn future incomes when they leave school. Individuals cannot support the public education privately, nor can they choose to receive the education from outside their own community.

Assume that within one community, the quality of public education is the same for every individual and it is equal to the expenditure per student within that community:

$$q_i = q(s_i) = s_i$$

And in the following part of the discussion, I refer "quality of public education" and "expenditures per student" as the same thing.

Given the population for one community, we can calculate the mean income $\bar{y}$ as well as the spending per student:

$$s_i = t_i \bar{y}_i$$

where t_i and $\bar{y}_i$ are the tax rate and mean income in community i.

In period two, individual's income is determined by the quality of public education in the community he/she resides:

$$y^2 = I\big(q(s)\big) = I(s)$$

the income function I is also strictly increasing, concave and twice continuously differentiable.

So the indirect utility function for a typical individual in the economy is given by:

$$V = u\big((1-t)y\big) + \beta u(t\bar{y})$$

β is the discount factor and there is no capital market, so the possibility that individuals can save for future consumption or borrow against future earnings is ruled out.

An individuals' preferred tax rate must satisfy $\frac{\partial V}{\partial t} = 0$, given $\bar{y}$:

$$u'\big((1-t)y\big) \cdot y = \beta u'(I) \cdot I(t\bar{y}) \cdot \bar{y}$$

Fernandez and Rogerson (1996) have proven the existence of initial equilibrium and provided its characteristics in multi-income-group and multi-community case. Those characteristics for the two-community and three-income-group case can be summarized as the following:

- ✧ The preferred tax rate by an individual is increasing in his/her initial income.

- ✧ The quality of public education in one community is increasing in the tax rate and the mean income within the community.
- ✧ y_1 individuals and y_3 individuals are separated in two communities, denote the community all y_1 individuals reside as C_1 (the rich community), the other community as C_2 (the poor community), and we must have that $(q_1, t_1) \gg (q_2, t_2)$
- ✧ y_2 individuals are indifferent between C_1 and C_2 (a low period one disposable income is compensated by a high quality of public education, or in other words, a high future income, vice versa), part of the y_2 individuals live in C_1 and the rest live in C_2

To simplify the analysis, I will focus on this two communities and three income groups (high, middle and low) case. Two levels of government (local and state) are introduced into the model, local governments collect tax revenues and fund the public education, the state government is the redistributive policy maker (as we shall see in the next section).

In order to make the Pareto-improving analysis tractable, two additional assumptions need to be made here:

ASSUMPTION 2.1:

The population for each income group is equal.

If the mass population is normalized to 1, we have:

$$P_1 = P_2 = P_3 = \frac{1}{3}$$

where P_j $(j = 1,2,3)$ is the mass population for income group j. Given $P_1 = P_2 = P_3 = 1/3$ and y_2 individuals are separated between C_1 and C_2 (this is the case under initial equilibrium), the majority voting results in the tax rates in C_1 and C_2 being chosen by y_1 and y_3 individuals respectively. And for the extreme case all y_2 individuals living in one community, one individual from the other income group in the same community will be the voter. With this assumption, the voters in both communities will not change even when migration happens.

ASSUMPTION 2.2:

$$\partial^2 u(I(t\bar{y}))/\partial t \partial \bar{y} = 0$$

Plug into the equation $\partial V/\partial t = 0$, the above formula is equivalent to $\partial t/\partial \bar{y} = 0$. This assumption suggests that individuals' preferred tax rate is independent with the mean income of the community, facing with the change of $\bar{y}$, individuals will only adjust their future consumption.

With these two additional assumptions, we can go further and discover other features of the initial equilibrium.

Suppose under the initial equilibrium, a fraction of λ[7] y_2 individuals live in C_1 and rest live in C_2, so the population of individuals living in C_1 is $\frac{1}{3}$ y_1 individuals and $\frac{\lambda}{3}$ y_2 individuals:

$$N_1 = \frac{1+\lambda}{3}$$

In C_2, the total population is constituted by $\frac{1}{3}$ y_3 individuals and $\frac{1-\lambda}{3}$ y_2 individuals:

$$N_2 = \frac{2-\lambda}{3}$$

Total tax revenue collected in C_1 is given by:

$$T_1 = \frac{1}{3}y_1t_1 + \frac{\lambda}{3}y_2t_1$$

And the per student spending in community one:

$$s_1 = \frac{\frac{1}{3}y_1t_1 + \frac{\lambda}{3}y_2t_1}{(1+\lambda)/3} = \frac{(y_1+\lambda y_2)t_1}{1+\lambda}$$

Similarly, we can calculate the per student spending in C_2:

$$s_2 = \frac{(y_3+(1-\lambda)y_2)t_2}{2-\lambda}$$

Plug these per student expenditures into the individual's indirect utility function, we can get the indirect utility function for each income group under the initial equilibrium, the disposable income for y_1 individuals, y_2 individuals living in C_1, y_2 individuals living in C_2 and y_3 individuals are $(1-t_1)y_1$, $(1-t_1)y_2$, $(1-t_2)y_2$ and $(1-t_2)y_3$ respectively, thus:

$$V_1 = u\big((1-t_1)y_1\big) + \beta u(I(\frac{(y_1+\lambda y_2)t_1}{1+\lambda}))$$

[7] Note that in the initial equilibrium, it cannot be guaranteed that y_2 individuals are evenly separated between the two communities. To find out the how many y_2 individuals living in C_1 in the initial equilibrium, we need the utility functional forms which are not included in this chapter, so without losing generality, I denote the fraction of y_2 individuals living in C_1 as λ.

$$V_2^1 = u\big((1-t_1)y_2\big) + \beta u\big(I(s_1)\big); \quad V_2^2 = u\big((1-t_2)y_2\big) + \beta u(I(s_2))$$
$$V_3 = u\big((1-t_2)y_3\big) + \beta u(I(\frac{(y_3 + (1-\lambda)y_2)t_2}{2-\lambda}))$$

where V_2^1 is the indirect utility for y_2 individuals living in C_1 and V_2^2 is the indirect utility for y_2 individuals living in C_2, and since y_2 individuals are indifferent between C_1 and C_2, we have that $V_2^1 = V_2^2 \equiv V_2$.

The initial equilibrium can be illustrated in the following (note that the areas of the communities do not need to be the same):

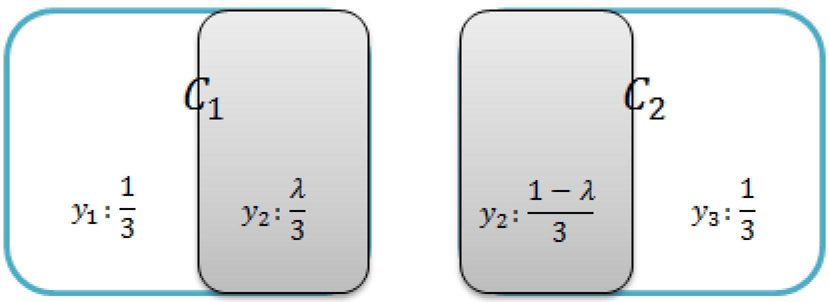

Figure 2.1: The initial equilibrium with two-community and three-income-group case

2.5 The Pareto-Improving Analysis

In the coming section, I am going to analyze the Pareto-improvement question based on one of the potential Pareto-improving policies introduced by Fernandez and Rogerson (1996):

"Another Pareto-Improving policy is to redistribute expenditures on education away from C_1 and toward C_2, for each t chosen in C_1, to redistribute a fraction γ of tax revenue in C_1 toward education expenditures in C_2……the outflow of y_2 individuals from C_1 to C_2 must be of a sufficient magnitude to reverse the fall in effective mean income in C_1 caused by this policy."

Once such a redistributive policy is applied, spending per student on public education decreases in C_1 and increases in C_2. All individuals in C_1 will be strictly worse off and all individuals in C_2 will be strictly better off. For individuals in C_1, y_2 individuals will for sure move to C_2 given that under the initial equilibrium $V_2^1 = V_2^2$ and now $V_2^{1\prime} < V_2^1 = V_2^2 < V_2^{2\prime}$. On the other side, y_1 individuals will stay in C_1 because moving to C_2 cannot guarantee they will be better off. As y_2 individuals leave C_1, the fall in effective mean income might be reversed by the increase in mean income.

To simplify the analysis, consider the extreme case under the redistributive policy, all y_2 individuals in C_1 have moved to C_2. Now I employ the comparative statics to solve the Pareto-improvement question mathematically.

The two states are: *STATE ONE*, the initial equilibrium; *STATE TWO*, after the redistributive policy is implied, all y_2 and y_3 individuals are living in C_2 and only y_1 individuals live in C_1. These two states are illustrated in Figure 2.1 and 2.2 respectively.

Now the mass population in each community is $1/3$ and $2/3$. The tax revenue in C_1 is $y_1 t_1/3$, suppose the redistributive fraction $\gamma = y_1 t_0/3$, the amount used for public education in C_1 is $y_1(t_1 - t_0)/3$, and in C_2, an additional amount of $y_1 t_0/3$ can be used on public education. We assume this redistributive policy is put forward by the state government.

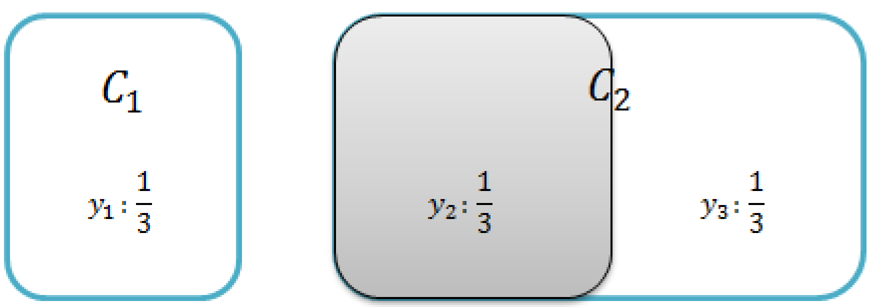

Figure 2.2: *STATE TWO* for the two-community and three-income-group case

C_1 is homogenous when all y_2 individuals moved to C_2, the expenditures per student is given as the following:

$$s_1' = y_1(t_1 - t_0)$$

Total education expenditures in C_2 is $\frac{1}{3}y_1t_0 + \frac{1}{3}y_3t_2 + \frac{1}{3}y_2t_2$, thus:

$$s_2' = \frac{\frac{1}{3}y_1t_0 + \frac{1}{3}y_3t_2 + \frac{1}{3}y_2t_2}{2/3} = \frac{y_1t_0}{2} + \frac{y_2 + y_3}{2}t_2$$

And the indirect utilities for each income group are as the following:

$$V_1' = u\big((1-t_1)y_1\big) + \beta u\Big(I\big(y_1(t_1 - t_0)\big)\Big)$$

$$V_2' = u\big((1-t_2)y_2\big) + \beta u\left(I\left(\frac{y_1t_0}{2} + \frac{y_2 + y_3}{2}t_2\right)\right)$$

$$V_3' = u\big((1-t_2)y_3\big) + \beta u\left(I\left(\frac{y_1t_0}{2} + \frac{y_2 + y_3}{2}t_2\right)\right)$$

Compare s_2' with s_2:

$$s_2' - s_2 = \frac{y_2 + y_3}{2}t_2 - \frac{(y_3 + (1-\lambda)y_2)t_2}{2-\lambda} + \frac{y_1t_0}{2} = \frac{y_2 + (1-\lambda)y_3}{2(2-\lambda)}t_2 + \frac{y_1t_0}{2} > 0$$

intuitively, s_2' is greater than s_2 because of the redistributive fraction $\frac{1}{3}y_1t_0$ and an increasing in mean income in C_2 caused by the migration of y_2 individuals. Given utility in period one remains constant for y_3 individuals and y_2 individuals in C_2, we have that $V_3' > V_3$ and $V_2' > V_2^2 = V_2$, so individuals from middle and low-income groups are strictly better off when the redistributive policy is implied.

Thus the redistributive policy is Pareto-improving ***if and only if*** $V_1' \geq V_1$:

$$u\big((1-t_1)y_1\big) + \beta u\Big(I\big(y_1(t_1 - t_0)\big)\Big) \geq u\big((1-t_1)y_1\big) + \beta u(I(\frac{(y_1 + \lambda y_2)t_1}{1+\lambda}))$$

$$\Leftrightarrow u\Big(I\Big(y_1(t_1 - t_0)\Big)\Big) \geq u(I(\frac{(y_1 + \lambda y_2)t_1}{1+\lambda}))$$

Since both u and I are strictly increasing, the above inequality is equivalent to:

$$y_1(t_1 - t_0) \geq \frac{(y_1 + \lambda y_2)t_1}{1+\lambda}$$

And the solution is:

$$t_0 \leq \frac{\lambda(y_1 - y_2)}{y_1(1+\lambda)}t_1$$

According to our redistributive policy, t_0 must be smaller than t_1, which is satisfied by the above solution, note that $y_1 - y_2 < y_1$ and $\lambda < 1 + \lambda$, thus $\frac{\lambda(y_1 - y_2)}{y_1(1+\lambda)} < 1$ and $t_0 < t_1$.

We also want *STATE TWO* to be stable, in other words, y_2 individuals in C_2 have no incentives to move back to C_1. Consider a marginal y_2 individual moves back to C_1, the impact of this migration on average education expenditures can be ignored, then this single individual's indirect utility is:

$$V_2^* = u\big((1 - t_1)y_2\big) + \beta u\left(I\big(y_1(t_1 - t_0)\big)\right)$$

thus to ensure the final state is stable, we need $V_2^* \leq V_2'$:

$$u\big((1 - t_1)y_2\big) + \beta u\left(I\big(y_1(t_1 - t_0)\big)\right) \leq u\big((1 - t_2)y_2\big) + \beta u\left(I\left(\frac{y_1 t_0}{2} + \frac{y_2 + y_3}{2} t_2\right)\right)$$

The above inequality gives the lower boundary of t_0, however without further assumption on functional forms, we cannot solve this boundary explicitly.

What if no $t_0 \in (0, \frac{\lambda(y_1 - y_2)}{y_1(1+\lambda)} t_1]$ exists such that $V_2^* \leq V_2'$ holds?

Government regulation can be used to ensure the final state is stable. Suppose the state government imposes an additional income tax T in C_1 and return the same amount only to y_1 individuals, so y_1 individuals are not affected while this T can prevent other individuals in C_2 move back to C_1 if T is large enough such that:

$$u\big((1 - t_1 - T)y_2\big) + \beta u\left(I\big(y_1(t_1 - t_0)\big)\right) \leq u\big((1 - t_2)y_2\big) + \beta u\left(I\left(\frac{y_1 t_0}{2} + \frac{y_2 + y_3}{2} t_2\right)\right)$$

So the Pareto-improving redistributive policy does exist. If the state government wants to subsidize the public education in the poor community as much as possible, it can just choose the fraction rate as $t_0 = \lambda(y_1 - y_2)t_1/[y_1(1 + \lambda)]$ and by doing so use a simple tax regulation to achieve the stability of the final state and ensure the realization of a Pareto-improvement.

2.6 The Model with Property Tax

In the previous section, I proved the existence of Pareto-improvement and provided the redistributive policy with public education financed by income tax. However in reality in the U.S., public education is generally funded by property tax revenue. Fernandez and Rogerson (1996) used income tax instead of property tax mainly because they prefer not to introduce housing market and want to focus the analysis based on a more transparent model (Fernandez and Rogerson, 1996). To capture the feature that most communities use property tax to determine the level of expenditure on public education, housing property tax is used to replace the income tax in the model. Additional assumptions also need to be added to the previous theoretical framework.

- A housing market is introduced to the model, to simplify the analysis, housing supply is assumed to be perfectly competitive. Since housing is the only private consumption in the model, h is the numeraire so $p_h = 1$.
- A housing property tax t (proportional to the housing value) is used locally to collect revenues and fund the local public education.
- Households, instead of individuals are now the agents in economy, each household consists of three members, parents and one child. Under this consumption, the number of households is the same as the number of students, so the spending per student on public education is equal to the education expenditure per household.

The two-community three-income-group model is still used here, within each income group, there are homogeneous households. Different household groups differ in their endowed income (or initial income) y_i $(i = 1,2,3)$, $y_1 > y_2 > y_3$ is also assumed here, and the mass population is normalized to 1.

There are two goods in the economy: housing (private consumption) and the public education, households gain utility from housing consumption and there exist certain technologies that transform spending on education into housing consumption.

We consider a two-period economy with all households having identical preferences described by the following:

$$u(h^1) + \beta u(h^2)$$

where h^1 is the household's housing consumption from period one (parents' housing consumption), h^2 is the housing consumption in period two (children's housing consumption) and β is the discount factor. Again, there is no capital market, so the households cannot save for future consumption or borrow against future earnings.

The interactions happen in the economy also follow a two-stage game. In the first stage, parents choose a community, the local government chooses the property tax rate determined by majority voting and the budget on public education. Based on the tax rate and the education budget (spending per student), parents decide whether to live in the current community or move to another one. Once all households are satisfied with the communities they choose and the economy is in its ***initial equilibrium***, they settle down, purchase the houses, pay the property tax and send their children to schools.

The period one housing consumption of a household with income y_j in community i would be:

$$h_{ij}^1 = \frac{y_i}{1 + t_i}$$

Suppose the average housing consumption in community i is $\bar{h}_i$, then spending per student in the community is given:

$$s_i = t_i \bar{h}_i$$

In period two, the young individuals enter the job market, earn income and purchase their own house, which is counted in the total household's utility.

Since $p_h = 1$ and there is no tax in period two, the amount of housing purchased in period two is equal to the future income:

$$h_i^2 = I\big(q(s_i)\big) = I(s_i)$$

the quality of public education within one community is equal to the per student spending in that community.

As we can see from the above equation, which community the children choose to live does not have any impact on period two's housing consumption. The only factor determines households' period-two utility is that in which community their children receive their education.

The indirect utility function for a typical household in the economy is given by:

$$V = u\left(\frac{y}{1+t}\right) + \beta u(I(t\bar{h}))$$

To discover the characteristics of the initial equilibrium under housing property tax, assumptions similar to Fernandez and Rogerson's (1996) model are made:

ASSUMPTION 2.3:

$$-\frac{u''(h)}{u'(h)} \cdot h > 2 \qquad \forall h$$

With higher period one housing consumption (initial income), households are willing to give up a larger fraction of the housing consumption to trade for a higher quality of public education (future housing consumption), and the trading ratio must satisfy the above inequality.

ASSUMPTION 2.4:

$$v_2^{1^*} = u\left(\frac{y_2}{1+t_1}\right) + \beta u\left(I\left(\frac{y_1 t_1}{1+t_1}\right)\right) > v_2^2 = u\left(\frac{y_2}{1+t_2}\right) + \beta u(I(\frac{y_2+y_3}{2(1+t_2)}t_2))$$

$$v_2^{2^*} = u\left(\frac{y_2}{1+t_2}\right) + \beta u\left(I\left(\frac{y_3 t_2}{1+t_2}\right)\right) > v_2^1 = u\left(\frac{y_2}{1+t_1}\right) + \beta u(I(\frac{y_2+y_1}{2(1+t_1)}t_1))$$

Assumption two works as the same restriction Fernandez and Rogerson (1996) made in their model that no community is homogeneous in the initial equilibrium. v_2^2 is the indirect utility for y_2 households if all y_2 and y_3 households living in one community and v_2^1 is the indirect utility for y_2 households if all y_2 and y_1 households living in one community. $v_2^{1^*}$ is the indirect utility for the marginal y_2 household if this household moves to the other community and lives with all y_1 households. $v_2^{2^*}$ is the indirect utility for the marginal y_2 household moving to live with all y_3 households. This assumption suggests that when all y_2 households live in one community, they always have the incentive to move to the other community.

In order to make the Pareto-improvement analysis tractable, two additional assumptions similar to income tax model are also made:

ASSUMPTION 2.5:

The number of households (population) for each income group is equal.

$P_j = \frac{1}{3}$ where P_j $(j = 1,2,3)$ is the mass population for income group j.

ASSUMPTION 2.6:

A household's preferred tax rate t *is independent of the mean housing consumption of community where he/she lives:* $\frac{\partial t}{\partial \bar{h}} = 0$

Assumption 2.5 and 2.6 ensure that the tax rates in both communities will not change if migration happens.

With these assumptions, now we can describe the characteristics of the initial equilibrium:

- ✧ The preferred tax rate by an individual is increasing in his/her initial income.
- ✧ The quality of public education in one community is increasing in the tax rate and the mean income within the community.
- ✧ y_1 households and y_3 households are separated in two communities, denote the community all y_1 households reside C_1 (the rich community), the other community C_2 (the poor community). We have $(q_1, t_1) \gg (q_2, t_2)$ and the poorest group in C_1 should have an income no less than that of the richest group in C_2.
- ✧ y_2 households are indifferent between C_1 and C_2 (a low period one housing consumption is compensated by a high quality of public education, or in other words, a high future consumption, vice versa). Some y_2 individuals live in C_1 and the rest live in C_2

Assuming in the initial equilibrium, a fraction of λ y_2 households live in C_1 and rest live in C_2. The total tax revenues collected in C_1 and C_2 are given as:

$$T_1 = \frac{y_1 + \lambda y_2}{3(1+t_1)} t_1$$
$$T_2 = \frac{y_3 + (1-\lambda) y_2}{3(1+t_2)} t_2$$

The population in C_1 and C_2 are $(1+\lambda)/3$ and $(2-\lambda)/3$ respectively, thus we can calculate the expenditures per student in each community:

$$s_1 = \frac{(y_1 + \lambda y_2)t_1}{(1+t_1)(1+\lambda)}$$
$$s_2 = \frac{(y_3 + (1-\lambda) y_2)t_2}{(1+t_2)(2-\lambda)}$$

The housing consumption for y_1 households, y_2 households living in C_1, y_2 households living in C_2 and y_3 households are $y_1/(1+t_1)$, $y_2/(1+t_1)$, $y_2/(1+t_2)$, and $y_3/(1+t_2)$ respectively, so the indirect utility functions for each income group are given as the following:

$$V_1 = u\left(\frac{y_1}{1+t_1}\right) + \beta u(I(\frac{(y_1 + \lambda y_2)t_1}{(1+t_1)(1+\lambda)}))$$
$$V_2 = V_2^1 = u\left(\frac{y_2}{1+t_1}\right) + \beta u\big(I(s_1)\big) = V_2^2 = u\left(\frac{y_2}{1+t_2}\right) + \beta u(I(s_2))$$
$$V_3 = u\left(\frac{y_3}{1+t_2}\right) + \beta u(I(\frac{(y_3 + (1-\lambda) y_2)t_2}{(1+t_2)(2-\lambda)}))$$

$V_2^1 = V_2^2$ because y_2 households are indifferent at the initial equilibrium.

To analyze the Pareto-improvement question, we use the same redistributive policy as well as the same comparative statics as we did in the income tax case: to redistribute a fraction γ of tax revenue in C_1 toward education expenditures in C_2. I start from the initial equilibrium and *STATE TWO* is when all y_2 households are living in C_2.

The redistributive fraction under property tax is:

$$\gamma = \frac{y_1 t_0}{3(1+t_1)}$$

In *STATE TWO*, the spending per student in each community is given:

$$s_1' = \frac{y_1(t_1 - t_0)}{1 + t_1}$$

$$s_2' = \frac{y_1 t_0}{2(1 + t_1)} + \frac{y_2 + y_3}{2(1 + t_2)} t_2$$

The indirect utilities when redistributive policy is implied, for each group of households:

$$V_1' = u\left(\frac{y_1}{1 + t_1}\right) + \beta u\left(I\left(\frac{y_1(t_1 - t_0)}{1 + t_1}\right)\right)$$

$$V_2' = u\left(\frac{y_2}{1 + t_2}\right) + \beta u\left(I\left(\frac{y_1 t_0}{2(1 + t_1)} + \frac{y_2 + y_3}{2(1 + t_2)} t_2\right)\right)$$

$$V_3' = u\left(\frac{y_3}{1 + t_2}\right) + \beta u\left(I\left(\frac{y_1 t_0}{2(1 + t_1)} + \frac{y_2 + y_3}{2(1 + t_2)} t_2\right)\right)$$

Similar to the case of an income tax, it is very easy to show that $V_2' > V_2$ and $V_3' > V_3$, and $V_1' \geq V_1$ guarantees the policy is Pareto-improving:

$$u\left(\frac{y_1}{1 + t_1}\right) + \beta u\left(I\left(\frac{(y_1 + \lambda y_2)t_1}{(1 + t_1)(1 + \lambda)}\right)\right) \leq u\left(\frac{y_1}{1 + t_1}\right) + \beta u\left(I\left(\frac{y_1(t_1 - t_0)}{1 + t_1}\right)\right)$$

The two types of taxes result in the same solution:

$$t_0 \leq \frac{\lambda(y_1 - y_2)}{y_1(1 + \lambda)} t_1$$

Without further assumptions on functional forms, we cannot determine the lower boundary of t_0 and whether the final state is stable or not. However, the state government can use a tax regulation to ensure the stability of the redistributive reform. Since households' initial incomes are given as exogenous, an income tax regulation is preferred than a property tax because it will not cause market distortion.

Imposing T in C_1 and returning Ty_1 only to y_1 households:

$$\frac{(1 - T)y_1}{1 + t_1} + \frac{Ty_1}{1 + t_1} = \frac{y_1}{1 + t_1}$$

the utility for y_1 households remains the same and if T is large enough, no household in C_2 has the incentive to move to C_1.

2.7 The Differences Between Two Taxes

Indirect utilities under income tax and property tax:

$$V = u((1 - t_i)y) + \beta u(I(s_i))$$

$$V = u\left(\frac{y}{1 + t_h}\right) + \beta u(I(s_h))$$

As we can see the two functions[8] follow a very similar form, however, we cannot compare between those two utilities or the corresponding components of those two functions, this is simply because we cannot use the same utility function u to measure individuals' or households' utility from income and housing consumption. However, we can make a comparison between average education expenditures under different taxes, see the table below:

Table 2.3: per student spending before redistribution under different taxes

Before Redistributive Policy	Income Tax	Property Tax
s_1	$\frac{(y_1 + \lambda y_2)t_1}{1 + \lambda}$	$\frac{(y_1 + \lambda y_2)t_1}{(1 + t_1)(1 + \lambda)}$
s_2	$\frac{(y_3 + (1 - \lambda)y_2)t_2}{2 - \lambda}$	$\frac{(y_3 + (1 - \lambda)y_2)t_2}{(1 + t_2)(2 - \lambda)}$

Table 2.4: per student spending after redistribution under different taxes

After Redistributive Policy	Income Tax	Property Tax
s_1	$y_1(t_1 - t_0)$	$\frac{y_1(t_1 - t_0)}{1 + t_1}$
s_2	$\frac{y_1 t_0}{2} + \frac{y_2 + y_3}{2} t_2$	$\frac{y_1 t_0}{2(1 + t_1)} + \frac{y_2 + y_3}{2(1 + t_2)} t_2$

[8] t_i, s_i denote the tax rate and education expenditures per student under the income tax, t_h, s_h denote that under the property tax rate.

Let $\Delta = s_1 - s_2$ be the gap between two communities' education expenditures per student, before redistributive policy is implied:

$$\Delta_{income\ tax} = \frac{(y_1 + \lambda y_2)t_1}{1+\lambda} - \frac{(y_3 + (1-\lambda)y_2)t_2}{2-\lambda}$$

$$\Delta_{property\ tax} = \frac{(y_1 + \lambda y_2)t_1}{(1+t_1)(1+\lambda)} - \frac{(y_3 + (1-\lambda)y_2)t_2}{(1+t_2)(2-\lambda)}$$

Since $1 + t_1 > 1 + t_2 > 1$, we have the inequality below:

$$\Delta_{income\ tax} > \frac{\Delta_{income\ tax}}{1+t_1} = \frac{(y_1 + \lambda y_2)t_1}{(1+t_1)(1+\lambda)} - \frac{(y_3 + (1-\lambda)y_2)t_2}{(1+t_1)(2-\lambda)} >$$
$$> \frac{(y_1 + \lambda y_2)t_1}{(1+t_1)(1+\lambda)} - \frac{(y_3 + (1-\lambda)y_2)t_2}{(1+t_2)(2-\lambda)} = \Delta_{property\ tax}$$

$\Delta_{income\ tax} > \Delta_{property\ tax}$[9] also holds when redistributive policy is implied.

On the other side, it is easy to see that $s_{income\ tax} > s_{property\ tax}$ for both communities whether there is a redistributive policy or not. These differences indicate that when other variables are the same (same income, same tax rate), housing property tax results in more equality while income tax results in more efficiency. The main reason for this difference is that incomes for individuals/households are assumed to be constant, the property tax will lead to less consumption on housing while income tax does not result in a decrease in labor supply. Thus total tax revenue collect under property tax is less than that under income tax ($\frac{ty}{1+t} < ty$).

2.8 Robustness Analysis

In the previous Pareto-improvement analysis, the mass population of each income group plays a very important role in determining whether a policy could be Pareto-improving or not: the population ratio in each community determines which income group is going to choose the tax rate. As population ratios change when migration happens, the tax rates in both communities might also change, which further results in changes in indirect utilities for all income groups.

[9] It is very likely we have different value of λ under different taxes, however, it is easy to show the inequality relations still hold.

With the assumption $P_1 = P_2 = P_3 = 1/3$, the tax rates are always determined by y_1 and y_3 individuals. Now, what if the population of each income group follows another distribution? (e.g., $\frac{1}{5}, \frac{3}{5}, \frac{1}{5}$)

The first thing to be clarified here is that without further assumptions on utility and income functional forms, it is impossible to determine the fraction of y_2 individuals living in C_1 in the initial equilibrium (this fraction is denoted as λ in the previous section), as we can see from the fact that y_2 individuals are indifferent between C_1 and C_2, $V_2^1 = V_2^2$:

$$\big((1-t_1)y_2\big) + \beta u\left(I\left(\frac{(y_1+\lambda y_2)t_1}{1+\lambda}\right)\right) = u\big((1-t_2)y_2\big) + \beta u(I(\frac{(y_3+(1-\lambda)y_2)t_2}{2-\lambda}))$$

(given $P_1 = P_2 = P_3 = \frac{1}{3}$), we cannot solve for λ explicitly.

Since the exact population of y_2 individuals living in C_1 and C_2 cannot be determined in the initial equilibrium, the median voter in each community is not clear. So it is useful to employ a general method to examine the robustness of the result. In the following, I will analyze for all income groups, how their indirect utilities vary according to the possible change of the tax rate in *STATE TWO*.

In C_1, from state one to state two, all y_2 individuals move out and only y_1 individuals are left, we have two possibilities:

1) In both states, a y_1 individual chooses the tax rate and t_1 is preferred by y_1 individuals, so this tax rate will not change as y_2 individuals move out. From the previous analysis, y_1 individuals retain their utility level when the redistributive policy is applied.
2) In state one, a y_2 individual chooses the tax rate. When all y_2 individuals are gone and the tax rate changes from t_1 (preferred by y_2 individuals) to T_1 (preferred by y_1 individuals), we should have that $V_1'(T_1) > V_1'(t_1) \geq V_1(t_1)$, and y_1 individuals are better off.

In C_2, we also have two possibilities:

1) The median voter does not change and t_2 is the tax rate in both states. From the previous analysis, both y_2 and y_2 individuals are better off

2) A y_3 individual chooses the tax rate in state one and a y_2 individual chooses the tax rate in state two. When all y_2 individuals in C_1 have moved to C_2 both groups in C_2 are better off, later a new tax rate T_2 (preferred by y_2 individuals) will be chosen to replace t_2 (preferred by y_3 individuals). This further benefits y_2 individuals, however, the total change in V_3 is ambiguous under T_2 because the first period's utility is going to decrease for y_3 individuals. If the increase in period two's income (caused by an increase in average education expenditures), adjusted for time discount, can reverse the fall in disposable income in period one, the policy is Pareto-improving, otherwise, it is not.

Consequently, for different population distributions, the policy might not be Pareto-improving. However, the local government can simply use a tax regulation to ensure the final result is Pareto-improving: if some individuals are worse off under the new tax rate, it will not pass the bill to change the tax rate.

Now, if we have m communities, what would be the (potentially) Pareto-improving policy?

The state government can take some of the tax revenue from the richest community C_1 and use it to subsidize the public education in the poorest community C_m. As the average education expenditures increase in C_m, it becomes more attractive to the boundary group between C_m and C_{m-1}, so those residents will move to C_{m-1}. This will result in an increase in the mean income in C_{m-1}. As such, the boundary group between C_{m-1} and C_{m-2} will move to C_{m-1}. This migration pattern continues. Finally, the boundary group between C_1 and C_2 will move to C_2. If the magnitude of migration is large enough, it can reverse the fall in the average education expenditures in C_1 and the redistribution is Pareto-improving.

From the previous analysis, we can see that the only factor that can reverse the fall in the average education expenditures in the rich communities is the rise in mean income, which can only be led by the migration pattern "from wealthy communities to less wealthy ones". Thus, such a migration pattern is the necessary condition for Pareto-improvement. Any policy results in this migration pattern are potentially Pareto-improving.

2.9 Conclusion

In this chapter, I examined one tax reform under the theoretical framework of multi-community with education as the public good supplied. In the initial equilibrium of my model, the individuals/households locate themselves in the communities according to their initial income: the high-income group and part of the middle-income group reside in one community (which is the rich one) and the rest middle-income group and the low-income group reside in the poor community. Both the tax rate and education expenditures per student are higher in the rich community.

The model is built simple enough to solve the Pareto-improvement question analytically, yet captures some important features people are concerned about in analyzing the provision of local K-12 public education: interactions between different income groups and different communities, government decisions on determining the local tax rate and education financing.

Unlike Fernandez and Rogerson (1996) who briefly discussed different reforms, including a tax cap, minimum spending requirement, income redistribution, expenditure redistribution, etc. I focused on education expenditure redistribution[10] and provided mathematical reasoning to show that this reform is Pareto-improving and should be taken into consideration when a reform on public education is to be put forward.

This chapter made contributions to the study of public education reform, which is the major concern of growing literature and policy making. The results of my analysis suggested that the mixed financing system with correct redistributive policy and government regulation could strike a balance between efficiency and equality in the provision of public education. The total social welfare for all residents (measured in indirect utility function) and per-student spending in the poor

[10] Compared with other reforms presented by Fernandez and Rogerson (e.g., to redistribute income), to redistribute education expenditures is more easy for state government to handle with and also leads to less distortion in the economy.

community increase while both average education expenditures and total welfare in the rich community can at least remain constant level. This is because the redistributive policy makes the poor community more attractive for the middle-income group who live in the rich community, so they will move to the poor community, which results in average tax base increasing in both communities. This increase in the average tax base in rich community reverses the fall in average education expenditures and makes the policy Pareto-improving.

The policy implication suggested here is that to reduce the gap in public education expenditures across different school districts, the government can use the correct education funding policy under the mixed financing system: to redistribute a certain amount of tax revenues used for public education from the rich communities toward the poor communities. In doing so, the funding efficiency loss can be avoided (or minimized).

Meanwhile, some interesting factors are not included in the model which can be used for future research directions. Among those, the most important three are: ① Different distribution of the population. To simplify the analysis, I assumed the mass population for all the income groups is equal so the median voter won't change as the middle-income group migrates from one community to the other. In the robustness analysis, I also showed that with a different population distribution, the redistributive policy may lead to different results. Thus, one of the future research directions would be to examine the case in which we have continuum individuals or households in the economy. In other words, the initial incomes for all individuals/households follow a continuous distribution (e.g., normal distribution). ② A more general housing market. The housing supply here is assumed to be perfectly competitive and the prices are exogenously determined. In a more general case, we could employ a housing market with upward-sloping supply curve and the housing price is affected by the property tax (so Assumption 2.6 in section 2.6 no longer holds in a general housing market). ③ Impact of income tax on initial incomes. In the comparison between income tax case and housing property tax case, one of the main reasons that income tax leads to more efficiency is that I took the initial incomes as given so there is no labor market distortion caused by income tax. However, in reality, income tax will result in distortion in the labor market and affect individuals/households' real income. So if we want to do a complete comparison between the effects of these two taxes, the distortionary impact of income tax must be taken into consideration.

In Chapter 3, I add a competitive housing market and population with real income distribution into my model, I focus on property taxation over income taxation to better approximate real world education finance and use quantitative methods to evaluate the reform posed in this chapter.

CHAPTER 3

EDUCATION FINANCING REFORM UNDER A GENERAL HOUSING MARKET: NUMERICAL SIMULATIONS

3.1 Introduction

In the U.S., among all methods of K-12 schooling (public education, private education, and home schooling), public education is dominant. According to National Center for Education Statistics (2009)[11], up to 88.7% of the students in grade 1-12 are attending public schools in 2007. Without a doubt, given the fact that most parents are choosing public schools for their children, the quality of public education will have a huge impact on our future generations.

Perhaps one of the most striking features of public education in the U.S. is the great disparity in per student spending across districts. One of example is illustrated in Table 3.1, average local funding in one school district can be more than four times as large as that in another district even in the same county. Although the state funding generally plays a redistributive role in the case study of Colorado presented here (in Table 3.1, districts with higher local tax revenue per pupil tend to receive less state tax revenue per pupil[12], however, there is an exception: Agate district in Elbert County), the gap in total funding per student is still large (total tax revenue per student in Agate is more than 15,000 while the number for Elizabeth is only 7,678).

The facts in Table 3.1 have attracted the attentions of not only economists but also sociologists and educationists. Fierce debates rise about the current situation and whether the state government should reduce the great disparity in average education expenditures across school districts.

Some sociologists and educationists (McClure, Wiener, Roza, and Hill 2008) suggest that actions need to be taken in order to further reduce the spending disparity across districts. Berg et al (2011) argued that a poverty trap may emerge under current education financing system. They

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[11] This is the latest available data.

[12] I will use LTP as local tax revenue per pupil and STP as state tax revenue per pupil, both notations are consistent with those in Chapter 4.

point out that poor income groups cannot afford to live in districts with good public schools. While students from good school districts are more likely to enter good universities or colleges and find well-paid jobs, many young people from the bottom income groups have to look for jobs without a bachelor's degree, most of them constitute the poor group of their own generation.

Table 3.1: Example of per student spending in Colorado, year 2010[a]

Elbert County, CO			Weld County, CO		
District	LTP	STP	District	LTP	STP
Agate	4,326	10,708	Pawnee	16,857	0
Elizabeth	2,637	5,041	Platte Re-7	8,980	1,408
Elbert	1,723	8,901	Ault Re-9	4,969	4,276
Big Sandy	1,353	8,347	Prairie	3,355	9,397

Source: Colorado Department of Education (CDE)

[a] Data are for elementary and secondary public schools. Since in this chapter I only focus on the study of Colorado, so Table 3.1 presents some data in Colorado, facts for other states can be found in Table 2.2, Chapter 2.

On the other side, economists take a very cautious attitude toward the further reform advocated by sociologists and educationists. Downes and Schoeman (1997) argued that the whole public education sector may be hurt by the further reform, if the rich households have noticed that the per student spending in their own communities has decreased significantly due to redistributive policies, they will choose private schools instead of public schools, which further results in the education resources (e.g., teachers, state and/or federal policies) flowing from public sector to private sector. Fernandez and Rogerson (1999) also found efficiency loss in the education financing system, because the number of rich school districts is far less than the number of poor school districts. Under further reform, it is very likely we have to face such a situation: rich districts have to bear significant fall in education expenditures while the additional amount of money each poor district receives is not enough to pull up the per-student spending to a desirable level.

To the best of my knowledge, Fernandez and Rogerson (1996) are the first to build a theoretical model to examine the provision of local public education under current financing system (a system where both local and state governments participate in, it is also called a mixed financing system). The model is simple enough yet captures all the major factors people might concern with: different income groups, a number of communities for individuals/households to choose, availability of migration and locally provided education. Fernandez and Rogerson (1996) focused on studying the equilibrium characteristics of the model and found that individuals stratify themselves into communities based on their initial income and community with higher mean income characterized by both higher tax rate and per-student spending on public education.

Fernandez and Rogerson (1996) also presented various reforms that would be Pareto-improving. According to their analysis, any policy results in an increase the population of the relatively high-income group in a relatively poor community is potentially Pareto-improving. In Chapter 2, I analytically show that in the three-income-group and two-community model, with additional assumptions on population distribution and individuals' preferences, the reform "to redistribute a fraction of education expenditures away from the rich community toward the poor community" is Pareto-improving.

The result can provide people some confidence on current education financing system. However, the model in the previous chapter is not complete, the housing market is omitted and local public education is funded by an income tax. Given the fact that local public education is mainly financed by housing property tax revenues, we would like to know when the housing market is introduced into the model and public education is financed by a property tax, whether the reform posed in Chapter 2 is still Pareto-improving or not.

The main contribution of this chapter is the use of numerical methods to illustrate that in a more complicated model with a general housing market and local public education financed by a property tax rate, the reform policy is still Pareto-improving under a two-community and three-income-group[13] case. The Pareto-improving reform works if the redistributive amount is correctly chosen by the state government.

The intuition behind the Pareto-improvement is that, when households move from the rich community to the poor community, housing demand increases in the poor community, causes housing price rise and a fall in utility from housing and private consumption. On the other hand, utility from receiving education increases as the state government subsidizes public schools in the poor community. When the amount of subsidy is large enough, the total effect of redistribution on the poor community is positive and at least some households in the poor community are better off. In the rich community, a decrease in total population causes a decrease in housing demand and utility from consumption increases. The maximum education expenditures the rich community can afford to be taken away (so that no household is worse off) is greater than the minimum education expenditures the poor community needs to receive, thus a Pareto-improving redistribution always exists.

The simulation model can also be used in a real world application, by employing county migration data, we can calculate the maximum decrease in education expenditures for counties with negative migration (or the minimum increase in education expenditures for counties with positive migration). By comparing with real education expenditure change data, we can tell the welfare change of given counties or use the results as a guidance when state financing goal is to make a Pareto-improvement.

3.2 Research Plan

The goal of this chapter is to examine whether the Pareto-improving reform mentioned in Chapter 2, to redistribute a fraction of education expenditures away from the rich community toward the poor community, will work in a more realistic world with public education is funded

[13] In this chapter, income is calibrated to the state context.

by a property tax posed on a general competitive housing market (with an upward sloping supply curve) and income groups generated by real world data.

The theoretical framework partly follows Fernandez and Rogerson (1998). They built an overlapping generations model to study the welfare change when the education financing system turned from local financing to pure state financing. In their model, Fernandez and Rogerson include all major factors to examine households' welfare: communities, income groups, private consumption, housing consumption, property tax, migration and the provision of local public education.

I use the households' preferences proposed by Fernandez and Rogerson, however, instead of an overlapping population with infinite periods, a two-stage game is employed in my model[14]. In stage one, households choose a community, vote for the property tax rate (the tax rate is determined by majority voting) and decide whether to stay or move to another community based on utility maximization until no household is strictly better off by moving to another community. In stage two, households pay tax, receive public education and make housing and private consumption decisions (In stage one, households make moving decisions and in stage two, they make consumption decisions).

Since the model is too complicated to solve analytically, I employ numerical methods. Simulations are run for Colorado in the year 1999, 2005 and 2008. To be consistent with the theoretical model in Chapter 2, the state is divided into two big communities, rich and poor, and all the income levels are sorted into three groups, low, middle and high, with the mass population of each equals to 1/3. Results show that when the redistributive reform is used and all middle-income households moved to the poor community, if the state government retains education expenditures in the rich community, the policy fails; if the state government maximizes the redistributive amount and retains the rich income group's utility level, the policy is Pareto-

[14] In Fernandez and Rogerson 1998 paper, they created future income as a function of quality of public education and a random shock, with this income function, Fernandez and Rogerson and deal with generations in infinite periods (if the results converge to a steady state). In my Chapter 3, I do not use an income function and I also want the model to be more consistent with the theory in Chapter 2, so a two-stage game is employed.

improving. Thus, the key factor determines whether the policy works or not is the redistributive amount chosen by the state government.

Given different migration rates, I also use the simulation model to solve: ① the minimum average education expenditures a poor community must receive and ② the maximum average education expenditures that the state government can take away from a rich community to ensure a Pareto-improvement. By looking into the migration and education expenditure data for major counties in Colorado, it is straightforward to tell which counties are better off, and which counties are worse off.

The simulations also uncover an irregular relationship between the migration rate and the minimum amount that has been received/ maximum amount been taken for one community to ensure a Pareto-improvement (for the poor community, the curve is an upward wave shape). Further study shows that this is due to the nature of the total utility function: $U = u(c, h) + v(q)$, where the first part, the utility from housing and private consumption is a transform of constant-elasticity-of-substitution function, and the second part $v(q)$, the utility from receiving education, is totally separated from $u(c, h)$. If the total utility function is a Cobb-Douglas form instead, the minimum amount a poor community must receive on public education to ensure a Pareto-improvement is an increasing function of migration rate (with second derivative less than zero).

3.3 Literature Review

The fundamental theory of my paper can be traced back to Tiebout (1956), *A Pure Theory of Local Expenditures*. Tiebout presented assumptions on how local public goods are provided in a multi-community world. Some of those assumptions also used in my model are: ① There is no migration cost so individuals (households) can freely move among communities in order to maximize their utility. ② Individuals or households have perfect information and can response to all policy change immediately. ③ All individuals or households are endowed with a certain amount of initial income. ④ One community's local public goods have no externalities on other communities.

My theoretical framework directly follows Fernandez and Rogerson (1996, 1997 and 1998). So far as I know, they are the first to examine the provision of local public education under a theoretical model which captures most major factors need to be taken into consideration: different income groups and communities, migration and locally provided public goods (public education).

In their first paper, Fernandez and Rogerson focused on the equilibrium of the model under a two-community and three-income-group case. In the equilibrium, individuals stratify themselves into communities based on their initial income. The rich group and the poor group are separated in different communities while the middle-income group (the boundary group) is indifferent between two communities. The rich community is characterized by a tax rate and quality of public education which are strictly greater that in the poor community. Fernandez and Rogerson also analyzed different reforms of public education and argued that any policy results in increasing the population of boundary group in the poor community is potentially Pareto-improving. This is because as boundary group increases in the poor community, mean income rises in both communities and causes the average education expenditures increase.

In my previous chapter, I examined one of the potentially Pareto-improving reforms posed by Fernandez and Rogerson (1996), "to redistribute a fraction of education expenditures away from the rich community and toward the poor community." Instead of using tax rate and average expenditures on public education, individual's total utility is employed as the only Pareto criterion[15], and by introducing additional assumptions on population distribution and individuals' preferences, I analytically showed that the reform is Pareto-improving.

Fernandez and Rogerson (1997) examined the zoning effect on the provision of local public education. A housing market is introduced into the model and public education is funded by a property tax and income follows a continuous distribution. The analysis suggested that when

[15] One criterion is easier to track in the Pareto-improving analysis, if both tax rate and per student spending are used as Pareto criteria, it is very likely we have the case that both tax rate and per student spending increase and the welfare change is ambiguous.

zoning is endogenized, the rich community is more exclusive since high-income individuals will choose a high zoning amount (the minimum housing consumption amount in order to live in the rich community), so the tax rate determined by majority voting is more preferred by high-income individuals. As more middle-income individuals are moving into the poor community, the gap between education expenditures may decrease because relatively more resources are spent on public education in the poor community. The welfare analysis showed that the poorest individuals are always worse off, this is because as more middle-income individuals move to the poor community, the tax rate chosen through majority voting is further away from the one preferred by bottom individuals.

In their 1998 paper, Fernandez and Rogerson built an overlapping generation model to examine the welfare change when the education financing system is turned from a pure local financing system to a pure state financing system. A two-community model is still used and total households are divided into nine groups based on the U.S. income census. To eliminate population change, each household is assumed to be consisted of one parent and one child. Households consume housing as renters. The new generation needs to rent their own houses when they enter the economy because old housing will perish completely with the old generation. Again, local public education is financed by a property tax on housing, households gain their utility from (private and housing) consumption and receiving the public education. Numerical methods are used to solve the problem, and the results suggested that when public education is purely supported by the state government across communities, per student spending is less than the rich community and greater than the poor community in the local financing system. One interesting result is that the total social welfare (based on total expected utility) in the state financing system is higher than that when public education is funded purely locally.

Recently, Calabrese, Epple, and Romano (2011) have done research similar to Fernandez and Rogerson (1998). They presented a model with heterogeneous households, general housing supply, migration, many communities and local public goods to examine the welfare effect in simulation model based on empirical evidence. The model is calibrated for both centralized and decentralized cases. In the centralized case, all communities are under regulation of one policy planner (state government, this is the state financing system in Fernandez and Rogerson (1998)). In decentralized case, each community has its own policy planner (local government, according to Fernandez and Rogerson (1998), this is the local financing system). The models in Calabrese, Epple and Romano (2011) and Fernandez and Rogerson (1998) share many similarities: ① Households are heterogeneous and can freely move among (between) communities. ② A property tax, determined by majority voting, is imposed on housing consumption and the tax revenues are used to finance the local public goods (public education). ③ Both models are built to examine the welfare change when a local financing system (decentralized case) turns into a state financing system (centralized case). The household's utility function and housing supply function Calabrese, Epple and Romano (2011) used in the model are different from those in Fernandez and Rogerson's 1998 paper. In addition, the income data they used are Metropolitan Area data from the 1999 American Housing Surveys (*AHS*) while Fernandez and Rogerson (1998) used 1980 National Census' data. The calibration results suggested that decentralization would lead to inefficiencies: compared with the centralized case, decentralization will result in more consumption distortion from tax, voting distortion in choosing the preferred tax rate, and jurisdictional externalities (where poor households tend to crowd richer communities). The last source of inefficiency (jurisdictional externalities) may go against one of the major predictions[16] that most literature use when multi-community feature is employed in the model: communities are stratified by income.

In order to run the simulations in this chapter, I also investigate literature that will help me decide parameter values. Cameron and Taber (2004) estimated education borrowing constraints by using returns of schooling, test scores are used as an approximation of quality of education and the income returns with respect to the quality of education can be obtained. Mayer and Somerville (2000) and Epple, Gordon, and Sieg (2010) examined the supply side of the housing market and found a price elasticity of housing supply greater than 3. Zabel (2004) did a survey on housing

[16] Fernandez and Rogerson derived the proposition of stratified communities in all of their 1996, 1997 and 1998 papers, this feature is further used to uncover other characteristics of the models.

demand literature, including the work of Mayo (1981) and Ermisch et al. (1996) and provided a range of price elasticity of housing demand to be (-0.5, -0.8).

3.4 The Model

This section presents the benchmark two-community and three-income-group model. The economy is populated by households with a total mass equal to one. Each household is consisted of two parents and one child, it can choose one of the two communities ($C_i \quad i = 1,2$) to live and is endowed with an initial income y_j ($j = 1,2,3$)[17] (without losing generality, we can assume that $y_1 > y_2 > y_3$), all decisions are made by households, each of whom has identical preferences:

$$u(c,h) + v(q)$$

Where u is the utility from private (c) and housing (h) consumption and v is the utility from receiving the public education (q), public education is funded by a proportional property tax on housing price (p). Both u and v are assumed to be concave, increasing in their arguments and twice continuously differentiable.

All households follow a two-stage game in the model. In stage one, each household chooses one of the two communities, vote for the property tax rate through majority voting and decide whether to stay in the current community or move to the other community based on utility maximization. This process will end when the economy reaches the ***Initial Equilibrium***.

DEFINITION 3.1:

The Initial Equilibrium is the situation in which none of the households are strictly better off by moving to another community.

PROPOSITION 3.1:

Given a community and a set of households as its residents, the majority voting results in the preferred tax rate of the household with the median income within the community.

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[17] i denotes the sequence number of communities so $i = 1,2$ and $j = 1,2,3$ is the sequence number of household groups.

In stage two, households make private and housing consumption decisions, pay property tax[18] imposed on housing and receive the public education. Again, public education in both communities cannot be funded privately. Households can only send their children to schools in their own community.

We also assume that within one community, the quality of public education q is the same for every pupil and define q equals the expenditures per student (or the average education expenditures) in that community:

$$q_i = e_i$$

When households in a community have made their housing consumption decisions, we can calculate the mean housing consumption amount $\bar{h}$ as well as the expenditures per student:

$$e_i = t_i p_i \bar{h}_i$$

where t_i, p_i and $\bar{h}_i$ are the tax rate, housing price and mean housing consumption in community i.

Each community has its own housing market with general housing supply function given by:

$$S = H^s(p_i)$$

Note that this supply function is the same for both communities, to simplify the model, we do not take the differences in land endowments and other factors into consideration.

The gross-of-tax housing price in community i is given by:

$$\pi_i = (1 + t_i)p_i$$

The households' preferences provided at the beginning of this section make this model tractable. Since the property tax rate is determined by majority voting and according to ***PROPOSITION 3.1***, all households in a given community can be divided into two categories: the voting household (the household with the median income) and the non-voting households (households whose initial incomes are not the median value in the community). For a typical non-

[18] The tax rate is determined by the "last voting" in stage one: in stage one, both communities will re-vote the tax rates after any household moving from one community to the other. When no household wants to move to another community after a voting, all households set down in their own communities, and the results (tax rates) from last voting will be used in stage two.

voting household, its problem is to choose the optimal combination of c and h to maximize its utility:

$$\max_{c,h} U = u(c,h) + v(q)$$

$$s.t. \qquad \pi h + c = y, \qquad c \geq 0, h \geq 0$$

the private consumption c is chosen as numeraire so its price is equal to 1. Given the maximization problem above, we can see that each community is characterized by the pair $(\pi_i, q_i) \quad i = 1,2$ from the perspective of households.

The major characteristics of stage-two can be summarized as the following:

- High-income households and low-income households are separated in two communities while middle-income households are indifferent between the two (part of them live in one community and the rest live in the other community). Denote the community with higher mean income as C_1 (the rich community) and the other as C_2 (the poor community).
- The gross-of-tax housing price and the quality of public education in C_1 are strictly greater than that in C_2: $(\pi_1, q_1) \gg (\pi_2, q_2)$.[19]

Now we turn to the property tax rates generated by majority voting, the preferred tax rate for the voting household with initial income y is determined by the following utility maximization problem:

$$\max_{t>0} u(y - \pi h, h) + v(q)$$

$$s.t. \qquad H^s(p) = N\bar{h}$$

$$q = tp\bar{h}$$

The first constraint is the housing market clear condition where N is the mass population of a given community. The second constraint is the local government budget balance condition: the average tax revenues equal to the education expenditures per pupil.

[19] The proof of these two properties can be seen in Fernandez and Rogerson (1998), *PROPOSITION 2*.

3.4.1 Functional Forms

The key to uncovering the impacts of the reform policy, "to redistribute a fraction of education expenditures away from the rich community toward to poor community," on all household groups, is to examine how U_j responds to the change of community population (N_i) and mean income $(\bar{y}_i)$. This is because the redistribution causes the population and mean income change in both communities, thus the households will adjust their consumption patterns and the preferred tax rates (only for the voting households), and finally leads to the change of their total utility.

The functional forms need to be specified are household's preferences on private and housing consumption $u(c,h)$, housing supply $H^s(p)$, and utility from receiving the public education $v(q)$. I use the functional forms that Fernandez and Rogerson employed in their 1998 paper:

$$u(c,h) = \frac{a_c c^\alpha + (1-a_c)h^\alpha}{\alpha} \qquad 0 < a_c < 1, \alpha \leq 1$$

$$H^s(p_i) = ap_i^b$$

$$v(q) = a_q\left[y_0 + \frac{B(1+q)^\delta}{\delta}\right] \qquad B > 0$$

Note that the specification of u is a transformation of a constant-elasticity-of-substitution (CES) utility function, housing supply function is constant elasticity and is identical for both communities, $v(q)$ is a normal concave, increasing and twice continuously differentiable function. a_c, α, a, b, a_q, y_0, δ and B are parameters.

3.4.2 Solving the Model

We start with the utility maximization problem for the non-voting households, plug the specifications of $u(c,h)$ and $v(q)$ into the total utility function:

$$\max_{c,h} \frac{a_c c^\alpha + (1-a_c)h^\alpha}{\alpha} + a_q\left[y_0 + \frac{B(1+q)^\delta}{\delta}\right]$$

$$s.t. \qquad \pi h + c = y, \qquad c \geq 0, h \geq 0$$

From the constraint we obtain $c = y - \pi h$ and the objective function yields to:

$$\max_{c} \frac{a_c(y-\pi h)^{\alpha} + (1-a_c)h^{\alpha}}{\alpha} + a_q\left[y_0 + \frac{B(1+q)^{\delta}}{\delta}\right]$$

By solving the first order necessary condition, we have:

$$h = \frac{y}{(\frac{\pi a_c}{1-a_c})^{\frac{1}{1-\alpha}} + \pi}$$

$$c = \frac{(\frac{\pi a_c}{1-a_c})^{\frac{1}{1-\alpha}}}{(\frac{\pi a_c}{1-a_c})^{\frac{1}{1-\alpha}} + \pi} \cdot y$$

Denote

$$g(\pi) = \frac{1}{\left(\frac{\pi a_c}{1-a_c}\right)^{\frac{1}{1-\alpha}} + \pi} = [\left(\frac{\pi a_c}{1-a_c}\right)^{\frac{1}{1-\alpha}} + \pi]^{-1}$$

And

$$g'(\pi) = -[(\frac{\pi a_c}{1-a_c})^{\frac{1}{1-\alpha}} + \pi]^{-2} \cdot \left[\left(\frac{a_c}{1-a_c}\right)^{\frac{1}{1-\alpha}} \cdot \frac{1}{1-\alpha}\pi^{\frac{\alpha}{1-\alpha}} + 1\right]$$

$$= -[g(\pi)]^2 \cdot \left[\left(\frac{a_c}{1-a_c}\right)^{\frac{1}{1-\alpha}} \cdot \frac{1}{1-\alpha}\pi^{\frac{\alpha}{1-\alpha}} + 1\right]$$

Households' private and housing consumption can be written as functions of their initial income:

$$h = g(\pi)y$$

$$c = [1 - \pi g(\pi)]y$$

Turn to the utility maximization problem for the voting households:

$$\max_{t>0} u(y - \pi h, h) + v(q)$$

$$s.t. \qquad H^s(p) = Ng(\pi)\bar{y}$$

$$q = tpg(\pi)\bar{y}$$

where $\pi = (1 + t)p$.

Totally differentiate the objective function yields the first order necessary condition:

$$u_c h = v'\bar{y}[g(\pi) + tp \cdot g'(\pi)]$$

By using the implicit function theory, we can obtain q_t from local government budget balance condition:

$$q_t = pg(\pi)\bar{y} + tp\bar{y} \cdot g'(\pi)\pi_t$$

Combined with $g'(\pi)$ and q_t, the explicit expression for first order necessary condition is given by the following:

$$a_c \cdot \left[\frac{\left(\frac{\pi a_c}{1-a_c}\right)^{\frac{1}{1-\alpha}}}{\left(\frac{\pi a_c}{1-a_c}\right)^{\frac{1}{1-\alpha}} + \pi}\right]^{\alpha-1} \cdot y^{\alpha}$$

$$= a_q B\bar{y}\left(1 + \frac{tp\bar{y}}{\left(\frac{\pi a_c}{1-a_c}\right)^{\frac{1}{1-\alpha}} + \pi}\right)^{\delta-1} \cdot [1 - tp \cdot \frac{\left(\frac{a_c}{1-a_c}\right)^{\frac{1}{1-\alpha}} \cdot \frac{1}{1-\alpha}\pi^{\frac{\alpha}{1-\alpha}} + 1}{\left(\frac{\pi a_c}{1-a_c}\right)^{\frac{1}{1-\alpha}} + \pi}]$$

Obviously, the maximization problem cannot be solved analytically, so I present numerical methods and analyze the reform policy in the next section.

3.5 The Simulations

3.5.1 Parameter Values

In this section, I report the parameter values I used in my simulations. For most of them, I use the ones in Fernandez and Rogerson (1998), meanwhile, I update some of the very important values: elasticity of housing supply (b), a_q[20], income distribution (y) and population distribution (N).

All the parameter values I will employ in my simulation are listed as the following:

Preference parameters:

$$a_c = 0.936, \alpha = -0.6, \delta = -3.9, B = 8, y_0 = 3.01$$

Housing supply parameters:

$$a = 1, b = 3.5$$ [21]

Population distribution (this is consistent with the assumption on population in Chapter 2):

$$P_1 = P_2 = P_3 = \frac{1}{3}$$

For income distribution, I investigate the income data in the state of Colorado provided by American Community Survey (ACS) in the year 1999, 2005 and 2008, all data are adjusted for inflation.

Table 3.2 provides all income and population distribution based on real world income data in the state of Colorado. The three income groups are generated by income data in ACS and mean income in the given years. It is assumed that in the ***Initial Equilibrium***, households with income equal to y_2 are indifferent between C_1 and C_2 (thus a_q is determined), they are split across the two communities (so y_2 households group is also called the *boundary households group*). So the mass populations of the two communities are equal $N_1 = N_2 = 0.5$, and we can further calculate the mean incomes for both communities.

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[20] Given the function form of $v(q)$, there is no specific economic interpretation on a_q.

[21] Fernandez and Rogerson (1998) provided a housing supply elasticity equal to 0.5, more recent literature tend to present this elasticity greater than 3 (Mayer and Somerville, 2000 and Epple, Gordon and Sieg, 2010).

Table 3.2: Income and population distribution, Colorado[a]

Year	1999	2005	2008
y_3	15	17	18
y_2	50	55	60
y_1	115	126	150
$\bar{y}_1$	93.33	102.33	120
$\bar{y}_2$	26.67	29.67	32
N_1	0.5	0.5	0.5
N_2	0.5	0.5	0.5
$\bar{y}$	60	66	76
a_q	0.051	0.063	0.081

[a] y_j is the income for household group j, $\bar{y}_i$ is the mean income for community i and $\bar{y}$ is the mean income for whole economy.

3.5.2 Results

In stage-two when median voting households have chosen the preferred tax rates for both communities, all households make their private and housing consumption decisions and receive the public education. By using numerical methods, the maximization problems posed in the previous part can be solved and the results are reported in the following table.

Table 3.3: Stage-two equilibrium values[a], Colorado

Year	1999	2005	2008
p_1	1.568	1.603	1.666
t_1	0.173	0.172	0.168
π_1	1.839	1.879	1.964
p_2	1.141	1.169	1.188
t_2	0.318	0.327	0.343
π_2	1.504	1.551	1.60
q_1	2.6192	2.8760	3.3408
q_2	1.1513	1.3207	1.4891
U_1	0.0259	0.0686	0.1344
U_2	-0.0563	-0.009	0.0546
U_3	-0.2729	-0.2072	-0.1409

[a] p_i, t_i, π_i, q_i are the housing price, tax rate, gross-of-tax housing price and per student spending in community i, U_j is the total utility for household group j.

As can be seen from the Table 3.3, $(\pi_1, q_1) \gg (\pi_2, q_2)$ is satisfied for all three years, housing prices (p_i), gross-of-tax housing prices (π_i), average education expenditures (q_i) and utility for all households groups (U_j) tend to be increasing functions in income (from the year 1999 to the year 2008, mean incomes and income for each households group are all increasing, this can be seen in Table 3.2). Spending per student is more than twice as large in the rich community (C_1) as in the poor community (C_2), this reflects one of the most important aspects of the current situation about public education in Colorado.

3.5.3 Policy Experiment

The policy I am going to examine in this section is "to redistribute a fraction of education expenditures away from the rich community toward the poor community." If the state government chooses a different redistributive amount, it is not surprising that the results of the redistribution would be different.

Before doing the policy test, I introduce *Stage-Three* in order to make the reform easy to analyze. This *Stage-Three* does not appear in Fernandez and Rogerson's paper, it is defined as the state when the state government has imposed the redistributive policy, migration has completed, new tax rates are voted and housing markets are cleared in both communities.

From *Stage-Two* to *Stage-Three*, the state government takes part of the education expenditures away from C_2 and use them to subsidize the public schools in C_2. Under the reform, all boundary households move to C_2 (here we first consider the case presented in Chapter 2: when redistribution is imposed and all middle-income individuals are living in C_2), then

- In C_1, a new tax rate is chosen by a y_1 household and consumption decisions are also made (since only y_1 group are left, the median voter can be any y_1 household). The state government fixes this new tax rate and takes a fraction of education expenditures away[22].
- C_2 receives half of the y_2 households and a certain amount of subsidy on public education, a new tax rate will be voted then new consumption decisions are made[23].

As a matter of fact, the state government can choose a fraction strictly greater than zero[24] and less or equal to the amount that makes the y_1 households at least as well off as in *Stage-Two* (because I want to examine whether the redistributive policy is Pareto-improving, making y_1

[22] Note by housing market clearing condition $H^s(p) = Ng(\pi)\bar{y}$, once tax rate is given, housing price is also given, and by the utility maximization problem of non-voting households, their consumption are also determined. So when state government fixed tax rate and takes education expenditures, households in C_1 will not change their consumption decisions.

[23] The population ratio for middle and low-income households are half-half, we can assume the median voter is still from low-income households or the state government assign a low-income household as the median voter.

[24] I assume the redistribution fraction strictly greater than zero so the state government can at least "do something"

households worse off does not make any sense). So the first policy I am going to test here is that *the state government takes education expenditures away from* C_1 *such that* y_1 *households retain their utility level*[25] (I define this as U_constant policy).

Table 3.4 reports the main results of the first policy for the year 2008. Results for the year 2005 and 1999 are included in appendices.

Table 3.4: U_constant redistribution, Colorado, year 2008

Year = 2008	Before the Reform	After the Reform
p_1	1.666	1.6
t_1	0.168	0.147
q_1	3.3408	2.5329
p_2	1.188	1.363
t_2	0.343	0.178
q_2	1.4891	1.6731
U_1	0.1344	0.1344
U_2	0.0546	0.0554
U_3	−0.1409	−0.1402

The simulations show that the U_constant policy is Pareto-improving for all three years, U_1 remains constant while U_2 and U_3 are strictly greater than before. p_1 decreases and p_2 increases mainly because 16.7% (1/6) of the total population have moved from C_1 to C_2, resulting in a shift in housing demand curves. Spending per student (q_1) decreases significantly for y_1 households due to the redistributive faction taken away by the state government, the fall in $v(q)$ is compensated by an increase in $u(c,h)$ caused by a drop in housing price. In C_2, as

[25] The redistributive fraction is maximized under this policy if the state government's goal is Pareto-improvement.

housing demand increases, housing price (p_2) rises, leading to the fall of utility from housing and private consumption for all households. However, with the increase in mean income and additional subsidy on local public education, the increase in $v(q)$ outweighs the decrease in $u(c,h)$, thus y_2 and y_3 households are all better off.

Table 3.5 reports the variable values for the poor community before and after the reform. A higher quality of public education is provided after the reform and the increase in utility from receiving the public education outweighs the fall in utility from housing and private consumption. Thus, both household groups are better off.

Table 3.5: Stage-two and Stage-three values, the poor community[a]

Year = 2008	Before the Reform	After the Reform
$u_2(c,h)$	-0.1845	-0.1846
$u_3(c,h)$	-0.3799	-0.3802
q_2	1.4891	1.6371
$v(q)$	0.2391	0.2400
$\bar{y}_2$	32	39
N_2	0.5	0.667

[a] q_2 is the total spending per student in C_2(including state subsidy if there is any), $u(c,h)$ differs between different households group because different household groups have different income, $v(q)$ is the same for every household within one community.

Since the policy with the maximum redistributive amount is examined, now I want to test a policy with a small redistributive amount, the choosing of such "a small amount" is rather arbitrary and there are as many choices as one can image. Yet the second policy I will test is that *the state government takes education expenditures away from* C_1 *such that* C_1 *retains its education expenditure level* (I define this as E_constant policy).

The results of the second policy are reported in Table 3.6. The same reform with a different redistributive fraction fails to be Pareto-improving this time, y_1 households are strictly better off while the rest two household-groups are worse off[26]. This is because the redistributive amount received by C_2 is not large enough so the increase in $v(q)$ cannot reverse the drop in $u(c,h)$ for both y_2 and y_3 households.

Table3.6: E_constant redistribution, Colorado, year 2008

Year = 2008	Before the Reform	After the Reform
p_1	1.666	1.6
t_1	0.168	0.147
q_1	3.3408	3.3408
p_2	1.188	1.349
t_2	0.343	0.254
q_2	1.4891	1.6226
U_1	0.1344	0.1351
U_2	0.0546	0.0542
U_3	−0.1409	−0.1425

From Table 3.7 we can see that under the U_constant policy, the redistributive fraction can lead to 0.5613 increase in average education expenditures in C_2, while the amount under the E_constant policy is only 0.1573. There is a huge gap between these two numbers so it is not surprising the U_constant policy is Pareto-improving and the E_constant policy is not.

Thus, we can conclude that with the given assumptions and theoretical framework, the key factor that determines whether the redistributive reform works or not is the redistributive fraction

[26] For the year 1999 and 2005, the results are the qualitatively similar.

chosen by the state government. It should be large enough (of course less than the maximum amount) so that all households in the poor community are better off. Provided with such conclusion, we can also find out the minimum fraction amount C_2 should receive to ensure a Pareto-improvement. It is the fraction amount such that one household group remains its utility level and the other is no worse off.

Table 3.7: State subsidies under different redistributive fractions

Year = 2008	U_constant	E_constant
Δq[a]	0.5613	0.1573

[a] Δq is the increase in average education expenditures caused by redistribution. The total increase in average expenditures is the result of both increase in mean income (income effect) and state subsidy (redistribution effect), Δq only captures the redistribution effect.

Table 3.8: Utility for households in C_2 under different redistributive amounts

Year = 2008	Before the Reform	After the Reform		
		$\Delta q = 0$	$\Delta q = 0.43$	$\Delta q = 0.5613$
U_2	0.0546	0.0537	0.0551	0.0554
U_3	-0.1409	-0.1434	-0.1409	-0.1402

As can be seen from Table 3.8, when all y_2 households have moved to C_2 and there is no redistribution, both groups are worse off. If each household in C_2 receives 0.43 subsidy on public education from state government, y_3 households' utility is unchanged while y_2 households are better off[27], thus $\Delta q = 0.43$ is the minimum amount every household in C_2 must receive to ensure the reform is Pareto-improving. For the state government, if it wants the reform to be Pareto-improving, the redistributive fraction should lie in the closed interval $[\Delta q_{min} \cdot N_2', \Delta q_{max} \cdot N_2']$[28].

[27] y_1 households are also strictly better off because the state government takes less education expenditures away from C_1 than it does under U_constant policy (0.43<0.5613)

[28] In 2008 Colorado case, $\Delta q_{min} = 0.43$; $\Delta q_{max} = 0.5613$, N_2' is the population in C_2 after migration and in this case $N_2' = 0.667$.

3.6 Welfare Change Analysis For Colorado

In the previous section, we have calculated redistribution fraction for a policy to be Pareto-improving, that is the maximum education expenditures the state government can take away from C_1 and the minimum education expenditures C_2 should receive so that no household is worse. These results are obtained under the condition "all boundary households in C_1 have moved to C_2,[29]" we can expect the redistribution fractions to be different under different migration rates.

Table 3.9 illustrates pairs of migration rates and redistributive amounts under different migration rates for both communities. Based on such feature, we can do a welfare change analysis for the counties in Colorado.

Table 3.9: Redistributive amounts under different migration rates

Community	Migration Rate	Redistributive Amount of q[a]
C_2	1.1%	0.011
	0.5%	0.007
C_1	-2.2%	-0.1628
	-0.6%	-0.0365

[a] For C_2, it is the minimum amount of q needs to be received to ensure no household is worse off, for C_1 it is the maximum amount of q the state government can take away and no household is worse off.

The data we need are county population and county-to-county migration in Colorado, these are provided in *American Community Survey* from *National Census Bureau*. The education expenditure data are collected from *Colorado Department of Education* (CDE), the original data are in school district level and I sort them into county level[30]. I use 2011 county-to-county migration data in Colorado and 2011-2012 education expenditure data, so it is assumed that the

[29] The migration rate is $= 33.3\%$ for both communities.

[30] If one school district covers an area belongs to different counties, its data will be counted proportionally in all counties it belongs to.

state government adjusts its current year education financing policy based on its previous year's migration condition.

How can we relate the simulation results with the real county data? First we need to match up the counties with the two communities, counties with positive net (county-to-county) migration are considered as the "poor community" and counties with negative net migration are considered as the "rich community" since in my model boundary households are moving from the rich community to the poor community under the state redistribution. For the poor community, it has an amount of average education expenditures before (denoted as E_{before}) the reform (also before the migration). When a reform is used and migration (real county-to-county migration rates are used in simulation) occurs, we calculate the minimum subsidy on public education the poor community should receive in order to ensure a Pareto-improvement and the total average education expenditures (including the state subsidy) after the migration (denoted as E_{after}).

Then the Pareto-improving expenditure growth rate is obtained by:

$$PI\ \ E\_Growth = \frac{E_{after} - E_{before}}{E_{before}} * 100\%$$

Once we have $PI\ \ E\_Growth$ under different migration rates, they are matched with counties with the corresponding migration rate. Then a comparison can be made between $PI\ \ E\_Growth$ and the real education expenditure growth rate for any given county. The similar process can be done for counties with positive net migration.

Table 3.10 reports the results for counties[31] with positive net county-to-county migration. Plug each county's migration rate into the simulation model we can obtain the Pareto-improving growth rate for the county, PI E_Growth. It is the minimum growth rate on average education expenditures to ensure that no household in the given county is worse off. When a county's real growth rate in average education expenditures (Real E_Growth) is greater than the growth rate

[31] I only provide the results for major counties (counties with large population), these counties are the "poor" community in the theoretical model.

under Pareto-improvement, all households in that county is better off. Otherwise, at least some households are worse off.

Table 3.10: Welfare change for counties with positive migration

County	Migration	PI E_Growth	Real E_Growth	Better Off
Adams	0.7%	0.712%	-4.17%	╳
Arapahoe	0.1%	0.853%	-4.9%	╳
Douglas	0.1%	0.853%	-2.27%	╳
Larimer	0.9%	1.034%	-2.3%	╳
Mesa	1.1%	1.357%	-4.97%	╳
Pueblo	0.5%	0.228%	-4.65%	╳
Weld	1.0%	0.5%	1.54%	○
Yuma	4.1%	2.24%	3.6%	○

From the table, we can see that among the eight major counties whose populations have increased, six of them are worse off. This is because all of the six counties have experienced negative growth in real education expenditures. Households in Weld and Yuma are better off since their growth rates in real education expenditures are greater than the growth rates under Pareto-improvement. Note that these two counties are also the poorest two in the state of Colorado, so the state government did a good job in helping the poorest two counties in the year 2012.

For counties whose net migration is negative (those counties are considered as C_1 in the simulation model because some households are moving out), we can also calculate the maximum decrease in average education expenditure (such that no household is worse off) and the real decrease in average education expenditure. The results are reported in the following table. Again, I only present the results for major counties. The results are reported in Table 3.11.

Obviously, most of the counties with negative net migration rate are better off since their real education expenditure decrease is less than the no worse off expenditure decrease. Eagle county's real expenditure has even increased during the year 2011 to 2012. The only county that is worse off is Boulder.

One striking feature from Table 3.10 and 3.11 is that the growth rate of Pareto-improving expenditure (or the decrease rate of no worse off expenditure) is not a monotonic function of the migration rate. People might think that as more households migrate to the poor community, more education resources are needed to be used on local public education to reverse the utility fall caused by the increase in housing price. As more money are spent on local public schools, the growth rate in education expenditures should also increase (the similar story can be told for the rich community as well, when households move out and housing price decreases, it can afford more education expenditure loss).

Table 3.11: Welfare change for counties with negative migration

County	Migration	NW E_Growth	Real E_Growth	Better Off
Boulder	-0.1%	-0.66%	-1.95%	×
Denver	-0.7%	-4.27%	-3.84%	○
Eagle	-0.5%	-0.46%	1.54%	○
El Paso	-0.3%	-3.9%	-2.58%	○
Jefferson	-0.3%	-3.9%	-3.32%	○

The feature reported in Table 3.10 and 3.11 is illustrated in Figure 3.1 and 3.2, although the curve follows an upward sloping trend in Figure 3.1 (a downward sloping trend for communities whose net county-to-county migrations are negative in Figure 3.2), it is not globally monotonic.

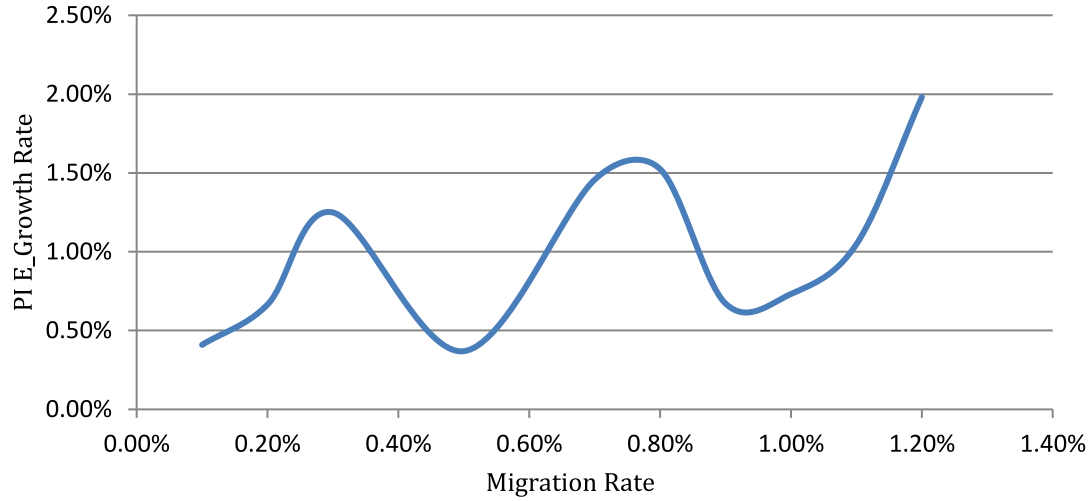

Figure 3.1: The relation between migration and the Pareto-improving growth in poor districts

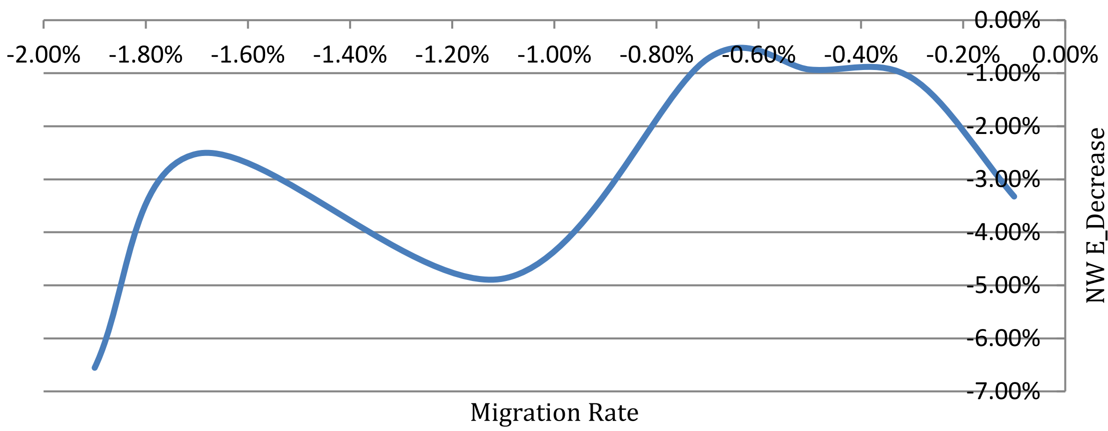

Figure 3.2: The relation between migration and the no-worse off decrease rate in rich districts

The main reason that we have non-monotonic relationship between migration and change in education expenditures (as shown in figure 3.1 and 3.2) is the nature of the utility function we choose to use in our theoretical model:

$$U = u(c,h) + v(q) = \frac{a_c c^{\alpha} + (1 - a_c)h^{\alpha}}{\alpha} + a_q \left[y_0 + \frac{B(1+q)^{\delta}}{\delta} \right]$$

The utility from receiving public education $v(q)$ is completely independent of the utility of private and housing consumption $u(c,h)$, which is a transformation of CES function. Thus when the exogenous conditions have changed (e.g., migration), c and h will move in the same direction while q might change at both directions because $v(q)$ is separated from $u(c,h)$.

We can use utility maximization to interpret Figure 3.1. When migration rate is between 0.1% and 0.3%, the marginal utility from receiving the public education v_{mar} is greater than the marginal utility from private and housing consumption u_{mar}, so the median voting household decides to vote higher tax rate and the per student spending continues to increase. Meanwhile, since both u'' and v'' are negative, v_{mar} decreases and u_{mar} increases[32] while migration rate increases from 0.1% to 0.3%. When migration rate is greater than 0.3%, $u_{mar} > v_{mar}$, households tend to consume more housing and private goods and the median voting household chooses a lower tax rate. As households prefer c and h to q, v_{mar} increases and u_{mar} decreases. This situation continues when migration rate reaches 0.5%, the households switch the consumption pattern again and do what they did when migration rate is between 0.1% and 0.3%. Such fluctuation will repeat and the whole curve follows an upward trend since more households move in means more resources (also more state government subsidy) need to be spent on public education. From an overall perspective, the large migration rate requires a greater increase in education expenditure to ensure no household is worse off. The story in Figure 3.2 is quite similar, the curve is a downward trend since more households move out, housing price drops more and the community can afford a greater decrease in education expenditures.

We can verify the irregular relationship between migration and Pareto-improving expenditure growth is due to the utility function form by employing a simple Cobb-Douglas function as utility function:

$$U = \alpha lnc + \beta lnh + \gamma lnq$$

[32] Since more households are moving into C_2 as migration rate increases, housing demand increases and h falls, so does u. $u'' < 0$, so u_{mar} increases.

The utility maximization problem for the non-voting households is:

$$\max_{c,h} \alpha lnc + \beta lnh + \gamma lnq$$

$$s.t \qquad \pi h + c = y, \pi = (1+t)p$$

The first order necessary condition yields:

$$\begin{cases} h = \dfrac{\beta}{(\alpha+\beta)\pi} y \\ c = \dfrac{\alpha}{\alpha+\beta} y \end{cases}$$

The average expenditures on public education is given by:

$$q = tp\bar{h} = \frac{\beta}{(\alpha+\beta)} \cdot \frac{t}{1+t} \cdot \bar{y}$$

Housing price can be solved from housing market clear condition:

$$p^b = N \cdot \bar{y} \cdot \frac{\beta}{(\alpha+\beta)p(1+t)}$$

$$p = [\frac{\beta N \bar{y}}{(\alpha+\beta)(1+t)}]^{\frac{1}{b+1}}$$

Suppose the community receives an average amount of A on public education from state government, the problem for median voting household is:

$$\max_{t} \quad \alpha ln \frac{\alpha}{\alpha+\beta} y + \beta ln \frac{\beta}{(\alpha+\beta)\pi} y + \gamma \ln(A+q)$$

Plug in $p = [\frac{\beta N \bar{y}}{(\alpha+\beta)(1+t)}]^{\frac{1}{b+1}}$ and drop the constant terms:

$$\max_{t} \quad -B\beta \ln(1+t) + \gamma \ln(A+q)$$

Where $B = \frac{b}{b+1}$ is a constant.

By solving the first order necessary condition, the community's preferred tax rate is:

$$t = \frac{\gamma \bar{y} - AB(\alpha+\beta)}{B[A(\alpha+\beta) + \beta \bar{y}]}$$

Thus, the total average education expenditures (including local funding and state subsidy) is:

$$q_{after} = q + A = D \cdot \bar{y} + C$$

$$D = \frac{\gamma\beta}{(\alpha+\beta)(B\beta+\gamma)}; \qquad C = A - \frac{AB\beta(\alpha+\beta)}{(\alpha+\beta)(B\beta+\gamma)}$$

Given the income groups $(18,60)$ and the corresponding population $(1/3, 1/6)$, if the migration is N_0 (the mass population moving from C_1 to C_2), mean income is given by:

$$\bar{y} = \frac{\frac{18}{3} + 60 * (\frac{1}{6} + N_0)}{0.5 + N_0} = 2 - \frac{28}{1 + 2N_0}$$

And we can write average education expenditures as a function of migration:

$$q_{after} = -\frac{28D}{1+2N_0} + 2D + C$$

which is an increasing, concave function with second derivative less than zero. The graph of q_{after} can be seen in Figure 3.3.

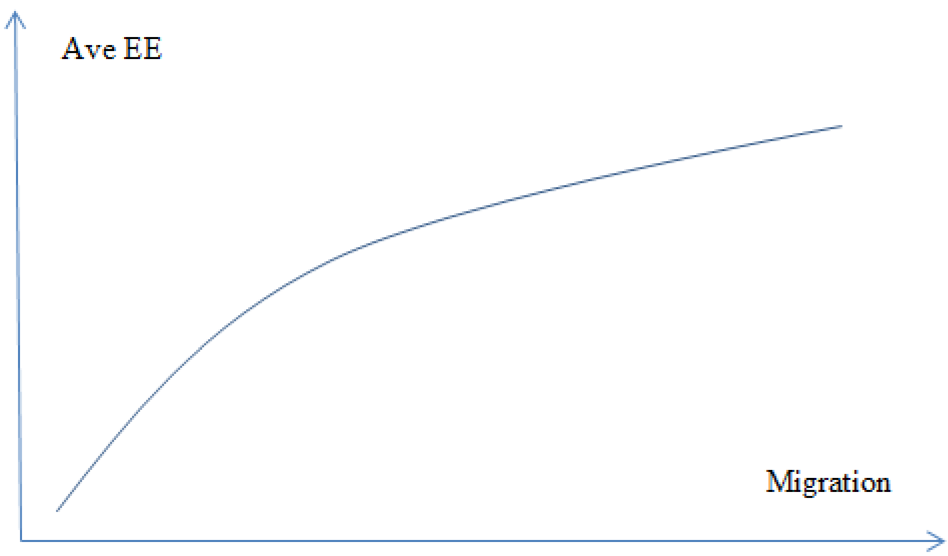

Figure 3.3: Relation between migration and average education expenditures (Ave EE) in the poor community Under a Cobb-Douglas utility function

3.7 Robustness Analysis

The simulation results rely on parameters obtained from empirical work, the change of even one of these parameters may totally change the simulation. However, it is not easy to match the economic data in the simulation with the observations in reality. This is because we have two communities, each is characterized by its own tax rate and housing price. So it is impossible to use one set of parameters that exactly fits both communities.

For many of the parameters, I use the values presented in Fernandez and Rogerson (1998), which is almost 20 years ago. I would like to know whether those old numbers, combined with some new ones (e.g., elasticity of housing supply, income distribution), produce reasonable results in my simulation model.

How to define a "reasonable result"? Suppose the ratio of housing expenditures to total consumption in C_1 is H/TC_1 and the ratio in C_2 is H/TC_2. If the real world data indicates a consumption ratio H/TC is between H/TC_1 and H/TC_2, we consider the simulation model creates a reasonable result. If the empirical studies provide a range of estimates, the simulation is considered as reasonable if the results for both communities fall in that range.

In the year 2008, the ratio of housing expenditures to total consumption in each community is provided as following:

$$\frac{H}{TC_1} = 0.1657; \qquad \frac{H}{TC_2} = 0.1357$$

The data from *U.S. Bureau of Labor Statistics, Consumer Expenditures in 2009* indicate that this ratio is 0.1417 in the year 2008.

From the year 2005 to 2008, the ratio of local spending on public elementary and secondary education to aggregate consumption expenditures (E/TC) has an average of 0.038[33], while the ratios for C_1 and C_2 are 0.0278 and 0.0465 respectively.

[33] Data provided by U.S. National Center for Education Statistics.

To choose a value for the returns of schooling, I rely on evidence provided by Stephen V. Cameron and Christopher Taber (2004). They used test scores (math, science and word) as an approximation of education quality to examine the returns of schooling on hourly wages. Their regressions showed that the returns of schooling can vary from as small as 0.02 (word score) to as large as 0.23 (science score), and in my simulation model, the return of education is 0.021 in the rich community and 0.137 in the poor community.

Jeffrey E. Zabel (2004) did a survey of the literature on the housing demand market, based on his survey I decide to match a price elasticity of housing demand ranges from -0.8 to -0.5. The elasticities in my simulation model are -0.6976 in the rich community and -0.6933 in the poor community. Mayer and Somerville (2000) and Epple, Gordon and Sieg, (2010) found housing supply elasticity higher than 3 and I use an elasticity to be 3.5 in my simulation in order to ensure we have solutions that the housing price is greater than 0 and the tax rate is between 0 and 1.

So although the parameters I choose to run my simulation may not reflect all the real world observations precisely, they are still within the acceptable range and the results produced can provide us insight about the education financing system in Colorado.

It is obvious that the change of parameter values may have impacts on whether the reform is more likely to be Pareto-improving or not. Among all the parameters I want to discuss two with explicit economic meanings: price elasticity of housing supply (b) and income distribution (y).

First, suppose the reform is applied in two different areas, Chicago and San Diego[34]. According to Green, Malpezzi, and Mayo (2005), the data from 1979 to 1996 show that the price elasticities of housing supply for Chicago and San Diego are 2.48 and 5.33 respectively. When the reform is applied and the boundary groups in both areas are living in the poor communities, the increase in housing price in C_{2c} is greater than that in C_{2s}, and the utility fall in C_{2c} is also greater than that in C_{2s}. So in order to make sure the reform is Pareto-improving, the government in Chicago needs to redistribute more education expenditures from its rich community to its poor

[34] In order to analyze the Pareto-improvement, each area is divided into two communities, C_{1c}, C_{2c} are the rich and poor communities in Chicago, C_{1S}, C_{2S} are the rich and poor communities in San Diego.

community. On the other hand, the drop in housing price in C_{1c} is greater than that in C_{1s} and the Chicago government can take away a larger amount of education expenditures from its rich community. Thus, without further calculation, it is hard to tell in which area the reform is more likely to work.

When the income inequality is high[35], the reform is less likely to fail. This is because the increases in mean incomes in both communities are higher under a higher income inequality (given the migration rate). In the poor community, more utility can be earned from public education (a higher mean income results in higher education expenditures). In the rich community, households can afford a larger amount to be taken away from education expenditures. Thus, the reform is more feasible in areas with a high income inequality.

3.8 Conclusion

In this chapter, I examined the tax reform posed in Chapter 2, "to redistribute a fraction of education expenditures away from the rich community toward the poor community," under a more realistic and more complicated model. A competitive housing market with upward sloping supply curve is introduced, local public education is funded by property tax revenues instead of income tax revenues.

There still two communities and all households are divided into three groups (high, middle, low) based on their initial income. Households follow a two-stage game when there is no state intervention. In *Stage-One*, each household chooses a community, votes for tax rate through majority voting, decides whether to stay in current community or move to the other community based on utility maximization until no household is strictly better off by migration. In *Stage-Two*, all households make private and housing consumption decisions, pay tax and receive the public education.

[35] Here a higher income inequality means that the population for each income group do not change while the differences between y_1 and y_2, and y_2 and y_3 are larger.

Since the model cannot be solved analytically, I employed numerical methods. The benchmark simulation model partly follows Fernandez and Rogerson (1998). They built an overlapping generations model to examine the reform of switching the education financing system from a local financing system to a pure state financing system. I used some of the functional forms from their paper in my simulation model, and the fundamental calibrations are based on Colorado income data for the year 1999, 2005 and 2008.

The fundamental simulation models indicate that under a more realistic and complicated model, the tax reform presented in Chapter 2 may still be Pareto-improving. Whether the reform works or not depends on the redistributive fraction the state government chooses, if the fraction is large enough, the reform works, otherwise it fails. This is because when the reform is imposed and the boundary households move to the poor community, increasing in total population leads to an increase in housing demand and a rise in housing price. A fall in private and housing consumption due to a higher housing price can only be reversed by increases in average education expenditures. Thus, only when the redistributive fraction is large enough such that the increase in utility from receiving education outweighs the fall in utility from consumption, the reform is Pareto-improving. From the fundamental simulation, we also know that the maximum redistributive fraction the rich community can afford to lose is greater than the minimum amount the poor community needs to receive (in order to ensure no household is worse off), thus if the distributive fraction lies in the correct range, the reform is always Pareto-improving.

When a simulation model is used to examine the state tax reform, migration is an exogenous variable. Further study shows that when migration rate changes, the maximum education expenditures the state government can take away from the rich community and the minimum education subsidy the poor community needs to receive (to ensure a Pareto-improvement) also change. This leads to the case study of the county welfare change in Colorado. By matching the real county-to-county migration rate, we obtained Pareto-improving expenditure growth rate ($PI\ \ E\_Growth$) for counties with positive net migration and non-worse off expenditure decrease rate ($NW\ \ E\_Decrease$) for counties with negative net migration. Then, we can tell which counties are better off and which counties are worse off by comparing $PI\ E\_Growth$ and $NW\ \ E\_Decrease$ with real expenditure changes.

The case study of Colorado also helps us to uncover one feature of the model: the relationship between change in $PI\ \ E\_Growth$ ($or\ \ NW\ \ E\_Decrease$) and migration rate is not monotonic within a small interval of given migration rate. This is because given the utility functional form, households have to make a tradeoff between education and private and housing consumption. When the marginal utility from receiving the public education is greater than the marginal utility of housing and private consumption, they vote for higher tax rate and $PI\ \ E\_Growth$ is an increasing function of migration rate, otherwise, $PI\ E\_Growth$ will decrease as more households migrate in. When the utility function is replaced by a Cobb-Douglas function, we have a regular positive relationship between $PI\ \ E\_Growth$ ($or\ \ NW\ \ E\_Decrease$) and the migration rate.

Finally, some other factors which are not fully examined can be used in further research, one of the most important factor among those is the number of communities. In this chapter, I assume there are only two communities, rich and poor. In the case study of Colorado counties, I divided major counties into two groups based on their net migration. One problem rises from the county of Douglas, it has one of the highest personal income per capita[36] in Colorado yet it is in the poor county group because its net migration is positive. Thus, the two-community model can provide us some insight about the education financing system in Colorado but we cannot tell much about the overall view of the state financing of public education due to the limitations of the model. In the next chapter, I do an empirical study based on all school districts in Colorado. The goal of the empirical study is to examine the Colorado state education financing policy with a global view and try to link the results from regression to (potential) Pareto-improvement we have discussed in Chapter 2 and Chapter 3.

[36] According to *American Community Survey*, Douglas' personal income per capita is 73,516 USD in year 2012

CHAPTER 4

IS STATE EDUCATION FINANCING POLICY POTENTIALLY PARETO-IMPROVING? A CASE STUDY OF COLORADO

4.1 Introduction

In the United States, although there are other choices for a child's education, including private schools and homeschooling, the majority of school-aged children are sent to public schools. According to the *National Center for Education Statistics* survey, since the year 2005, nearly 90%[37] of school-aged children attend public schools; this percentage has continued to increase since 2005.

Table 4.1: Example of per student spending in Colorado[a]

School District	Total Expenditures per Pupil		
	Year 2010	Year 2011	Year 2012
Agate	18,380	17,781	16,179
Archuleta	9,969	9,611	10,678
Briggsdale	18,261	17,288	17,494
Canon	9,557	9,000	8,648
Hinsdale	21,775	18,953	18,412
Kiowa	10,990	11,160	9,775

Source: Colorado Department of Education
[a] The data are for elementary and secondary schools.

During this period of time, there has been a growing disparity in primary and secondary public education across the country. Based on data provided by the Colorado Department of Education (DCE), spending per student on elementary and secondary education in rich school districts can be twice as much as the expenditures per pupil in the poor ones even within the same state (as can

[37] Source: *2013 National Center for Education Statistics*.

be seen in Table 4.1). This is one of the most striking features of the public education in the United States today.

It is not surprising that the gap in expenditures per student between the different school districts could be extremely large. According to *The Public School Finance Act of Colorado*, the disparity in average education expenditures across the districts largely reflects the disparity in the wealth conditions across those school districts[38].

Another well-known fact about public education is that in most school districts, public schools are funded under a mixed financing system where one school district's major education expenditures come from both local and state tax revenues[39]. Although the mixed financing system does reduce disparity in education expenditures (state government financing works as a redistribution[40] to certain extent), it receives criticism from both social activists and educationists.

The critics do not think that governments have done a good enough job (McClure et al., 2008) in reducing the disparity in public education spending across the different districts. They argue that under the current financing system, low-income families who cannot afford the living costs of rich school districts have to send their children to public schools in poor districts[41]. In this way, they

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[38] A district's preliminary per pupil funding is given by the following formula:
Preliminary Per Pupil Funding = [(Statewide Base * Personnel Costs Factor * Cost of Living Factor)
+(Statewide Base * Nonpersonnel Costs Factor)] * District Size Factor
Where statewide base is the same for every district in a given year and district size factor reflects purchasing power differences among districts. Note that this preliminary per pupil funding is not the final funding received by school districts.

[39] Although school districts do receive a certain amount of education tax revenues from the federal government, the percentage is tiny (For example, the federal grants count less than 10% of total expenditures per pupil in most school districts in Colorado). In this paper, when the "mixed financing system" is used, it refers to a mixture of local and state financing.

[40] Funding from the state is provided to each school district whose local share is insufficient to fully fund its Total Program: State Funding = Total Funding – Local Share
In Colorado, the local share can count as much as 80% of total funding for rich districts and as low as 20% for poor districts (data from DCE).

[41] Actually, in some states, parents are allowed to send their children to schools in districts for which they are not zoned. In Colorado, it is called School of Choice Act or Open School Enrollment. However, it is not easy for pupils to attend schools outside their own districts. The schools should have enough space and teaching staff, the students should meet the established criteria for school programs and the schools have no desegregation plan. In addition, students may even. likely lose free transportation to schools. According to the latest data from *National Center for Education Statistics*, in year 2007, among students who received public elementary and secondary education (88.7% of school-age children chose to attend public schools), 82.5% of them went to assigned schools. Thus the *School of Choice Program* does not go against the statement "households send their children to neighborhood schools" and it is reasonable to assume students cannot attend schools outside their own districts. We do not have data at school level, so in this chapter, the choice program within a school district is not considered.

receive a poorer quality education and are more likely to become low-income individuals in the future and continue struggling at the bottom of society, just as their parents did in the past (Berg et al., 2011).

On the other hand, some economists (Fernandez and Rogerson, 1999) have argued that further reform may hurt the efficiency of the financing system because education expenditures in rich districts will decrease greatly. What is worse is that further reform may hurt the public education sector as a whole (Downes and Schoeman, 1997). When rich households see the quality of public schools decrease, they might send their children to private schools. This will cause some of the education resources (teachers and certain funds) to flow into the private sector.

The debates over the past several decades have motivated researchers to aim at uncovering preferred reforms in currently used financing systems. Fernandez and Rogerson (1996) built a multi-community and multi-income-group model to examine the provision of local public education. In their paper, they discussed a variety of potentially Pareto-improving reforms. Chapter 2 of my dissertation directly follows Fernandez and Rogerson's theoretical framework. I choose one of the reforms and analytically prove that with additional assumptions, the reform "redistribute education expenditures away from the rich community toward the poor community" is Pareto-improving.

In the third chapter, a more complicated and more realistic model is built, a general housing market is employed and local public education is financed by a property tax imposed on housing consumption. Since the model cannot be solved analytically, numerical methods are used. The simulation results show that under a more realistic model with a housing market, the reform policy posed in Chapter 2 may still be Pareto-improving: if the redistributive fraction is large enough to reverse the utility fall caused by increase in housing demand in the poor community, and less than the maximum amount the rich community can afford (so that no households in the rich community is worse off), the reform works.

I analyze the models with only two communities in the previous two chapters since it is impossible to deal with a model with a large number of communities and track the Pareto-improving solutions. At the end of Chapter 2, I also derive the necessary condition of Pareto-improvement under a more general circumstance (there are a large number of communities in the economy): the migration pattern of people moving from rich communities to less rich communities.

In this chapter, I analyze the state financing policy in Colorado using an empirical approach. I use panel regressions to determine whether the Colorado state government is redistributing education resources away from the rich school districts toward the poor school districts and whether the current policy is potentially Pareto-improving.

The data was obtained from the *Colorado Department of Education* for the years 2010 to 2012. There are 179 districts in the count for each year and I have 537 total observations. Four regression models were developed to examine how state support on public education changes as local wealth varies. The regression results illustrate that the Colorado state government is playing a redistributive role in general, but under the current policy, residents in the less wealthy districts may want to move to the wealthier ones, and the poor districts are not attractive to those who live in the less poor districts. This does not satisfy the necessary condition of the Pareto-improvement derived in Chapter 2. Thus, it is recommended that the policy to be slightly changed.

The regression models illustrate that under current policy, state tax contribution per pupil (*STP*) is a decreasing function of local tax contribution per pupil (*LTP*) and the slope of the *LTP-STP* curve follows a flat-steep-flat pattern: the absolute value of the slope is smaller for the districts with a low or high *LTP* and greater for the districts in the middle. For all poor and rich districts and most of the districts in the middle, when local funding per student increased by one dollar, the state funding falls less than one dollar. So households from relatively less wealthy districts will move to wealthier ones, which is against the necessary condition of Pareto-improvement derived in Chapter 2.

Under the ideal policy, richer districts should lose more marginal state funding, as local wealth increases, and the state government needs to increase the marginal subsidy to the poor districts as their wealth decreases. The regression results show that when local tax revenue per student increased by one dollar, current policy decreases state tax revenue per student by 40 cents for rich and poor districts, and for most of the middle districts, average state funding is decreased by 80 cents. A potentially Pareto-improving policy is that the state government decreases *STP* by one dollar and 50 cents for all school districts when *LTP* increased by one dollar.

4.2 Research Plan

In this chapter, I am going to conduct an empirical analysis based on the data from the state of Colorado. There are two goals for this empirical study:

1. To test whether the state government is redistributing education resources from the rich school districts[42] towards the poor school districts.
2. If the answer to the previous question is "Yes," then I will examine whether the currently used redistribution policy is potentially Pareto-improving.

Four models are introduced to analyze the state financing policy on public education in Colorado. Model one is a simply regression model used to examine whether the state government is redistributing education expenditures away from the rich school districts toward the poor districts. Models two, three and four are used to examine whether the current policy satisfies the necessary condition of Pareto-improvement. In model two, dummy variables are used and all the school districts are divided into two groups: the rich group and the poor group. Since the result of model two does not show any evidence of nonlinearity, model three is introduced. In model three, square and cubic terms of the major independent variable (local tax contribution per pupil) are used. In addition to nonlinearity, the regression also indicates a "flat-steep-flat" pattern of the slope. In the last model, all school districts are divided into three groups, poor, middle and rich. The results in model four are consistent with that in model three: the coefficients of the poor and rich group are statistically equal (in the absolute values) and are significantly less than (the absolute

[42] In the following part, I use the school districts instead of the communities because the former one is more frequently used in reality; the two different notations actually mean the same thing in this chapter.

value of) the coefficient of the middle group, the tests of the coefficients also show that with significance level below 5.32%, the absolute value of the middle group is greater than 1.

At the end of this chapter, I analyze the current policy based on the results of four models, conclude that the policy does not satisfy the necessary condition of Pareto-improvement and present some suggestions.

4.3 Literature Review

The foundation of my theoretical framework is built on the discussions in Tiebout (1956), *A Pure Theory of Local Expenditures*. Tiebout made many assumptions on how local governments provide public goods in a world with multiple communities which have been widely used in the multi-community analysis. Two of them are also used in this chapter: ① There is no moving cost so households/individuals can move freely from one community to another. ② There are no externalities (either positive or negative) among communities.

In the empirical aspect, Feldstein (1975) built a log-linear regression model and found that the expenditure per pupil in one community is positively related to the wealth per pupil in that community. Fernandez and Rogerson (1997) used panel data for different states from 1950 to 1970 and modeled the impact of growth in personal income on per student expenditures on public K-12 education. Their results indicated that the average education expenditures tended to grow at nearly the same rate as the personal income per student.

The conclusion drawn from Corcoran and Evans (2010) is in contrast with much of the theoretical and empirical work. They suggested that income inequality decreases median voter's tax share, results in higher expenditures on public education. The estimation illustrates that 12% to 22% of the growth in local public education spending from 1970 to 2000 can be attributed to the increase in income inequality. The reason is that as the income inequality increased, median voter's tax share decreased, which resulted in higher local public education expenditures.

Evidence from those papers shows that personal income/wealth plays a key role in financing public education. Consequently, in my own research, I try to build regression models to examine the relationship between local wealth and the state financing policy.

Downes and Shah (2006) investigated the effect of school finance reforms on per student expenditures in the United States from the year 1970 to 1990 and pointed out that the local stringency of a reform matters in determining the results. In order to examine their argument, Downes and Shah divided observations (50 states, 20 years) into two groups based on whether the school finance reform is court-ordered or legislative. Then they built a regression model using log of average education expenditure in state as dependent variable, and stringency dummy (court-ordered or legislative) and other local factors (e.g., per capita income, median family income, the fraction of population between 5 and 17, the fraction of African-American population) as independent variables. The regression results show that the most of the coefficients in the court-ordered reform group are statistically different from the ones in the legislative reform group. Downes and Shah also found that the spending per student is lower in states with a larger faction of old population (people over 65), Hispanic or African-American population, the average state spending is higher when more shares of funding are provided by state aid and the number of districts per pupil is larger. To further model the finance structure change, the authors employed standard dummy variable framework in year (if the education finance reform took place in California in the year 1980, then this dummy is 0 for California from the year 1970 to 1979, the dummy value is 1 from the year 1980 to 1990). By introducing the interaction terms of this year dummy variable and other independent variables, the regression shows that the court-ordered reform appears to work immediately once such a reform is passed and will be almost completed within 5 years, while the transition associated with a legislative reform is much longer and it also takes many years to complete.

Clark (2003) conducted a research on *The Kentucky Education Reform Act (KERA,* passed into law in 1990*)*, the main goal of which is aimed at equalizing the average education expenditure across the school districts. In the regression model, Clark used the school finance outcome variable (state funding per pupil or local funding per pupil) as the dependent variable and the measurement of district's wealth as one of the major independent variables. The benchmark model presents a coefficient between state funding per pupil and district's median income equals to -8.68 for Kentucky. The coefficient for Tennessee is -5.42. When controls for district-level covariates (including district enrollment, whether the district is elementary or unified, whether the district is a county or a city) are included, the regression coefficients are -4.23 and 2.56 for Kentucky and Tennessee respectively. Clark concluded that KERA has effectively reduced the education expenditure gap across districts.

Baicker and Gordan (2004) examined the effect of mandated state education spending by comparing the regression results of states with *school finance equalization* (SFE) and states without SFE. SFE is defined as a dummy variable in the regression model with state expenditure as a dependent variable. The result shows that the effects of SFE are positive, indicating that school finance equalization is not locally self-financed but rely on state funding. Baicker and Gordan also ran the within-state regression to examine the relationship between local education expenditure (county level data, as dependent variable) and locally received state expenditure (as the independent variable). The coefficient suggests that every dollar increased in state funding will lead to local expenditure falls about 41 cents. By introducing locally received state expenditure as a function of local demographics and SFE (as the dummy variable), the relationship between local expenditure and SFE is estimated via two-stage least squares, counties are also divided into different groups based on their income conditions. The coefficient for SFE is negative, indicating that the reduction of education expenditure disparity is at the expense of drawing resources away from other programs (e.g., public transportation, medicaid), and counties with high-income are more likely to cut back on those programs.

Card and Payne (2002) did a research on the effects of local wealth on school finance reform (equalization of spending per student across school districts). Data are collected from 39 states from the mid-1970s to early 1990s. They built two models to examine how the change of local income (measured by the median family income in a district) would affect both state aid per student and total spending per student in a district. The first and second regressions used state aid per student and total spending per student as the respective dependent variable, independent variables include median family income and a vector of observations that affect school spending (e.g., the range of grades, whether a district is urban or rural).

The regression results show that richer districts tend to have a larger total spending per student but less average state aid. Districts from states with court decision (on equalization reform) tend to receive more state aid than those districts from states without court decision (the coefficients for median family income in the "average state aid-income" regression are larger in absolute values for the districts from states with court decision). Card and Payne then compared the changes in median income coefficients in the first two models by building the changes in coefficients as functions of a set of dummy variables for various reform events (whether the states dropped or added specific components in their education financing system). The estimation shows strong evidence of equalization under unconstitutional ruling (the term "unconstitutional" is defined as states with explicit and large changes in school finance system): if the income gradient of state aid falls, the income gradient of local total spending also decreases (note that the slope of the "state aid-income" regression is negative). This indicates the "state aid-income" curve is steeper and the "total spending-income" curve is flatter for the districts in the state with unconstitutional school finance system. Card and Payne also examined the impact of school reform on students' academic performances. They used samples of SAT scores from the same period and found that equalization of spending per student across school districts also narrowed the SAT scores of students with different family backgrounds.

In the above literature, the authors mainly focused on examining the impacts of various education financing reforms. Their research showed that those reforms did reduce the disparity in education expenditures among districts, which is consistent with my regression results that the state government plays a redistributive role in financing local public education.

4.4 Data and Description of the Regression Model

In Colorado, the major part of local public schools' funding comes from the local school districts and the state government. While local districts primarily use property tax to finance their schools[43], state expenditures on public education are collected from state income and sales taxes. Since the rich school districts have a high personal income per capita (PIPC) and the poor school districts have a low PIPC, the average tax revenue each district contributes to the state education fund is positively related to its wealth condition (measured in PIPC). So if the average education fund received by school districts is negatively correlated to their wealth condition, we can draw the conclusion that the state tax policy is playing a redistributive role.

The data used in this investigation was obtained from the *Colorado Department of Education* (CDE), from years 2010 to 2012. For each year there are 179 districts in the count[44], so in total I have 537 observations in my panel data. To capture the real effects of the data, the change in the dollar value must be excluded. As such, all expenditures in the year 2010 and 2011 were adjusted for the real values in the year 2012.[45]

In the following, I list all of the variables that will be used in the regressions:

LTP: local tax contribution per pupil, which is an approximation of local wealth[46].

STP: state tax contribution per pupil, which is used to measure the average funding received by the school districts.

FGP and ***SGP***: federal grants per pupil and state grants per pupil, respectively, these are the money received by the school districts from the federal and state governments for special education programs[47].

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43 According to *School Finance in Colorado* (a booklet prepared by Legislative Council Staff, 2015)

Local Share=Current Year Property Taxes + Prior Year Specific Ownership Taxes

The tax a Colorado resident pays every year when registering his/her car is specific ownership tax.

44 The actual number of school district in Colorado now is 178, there is one statewide "Charter School District."

45 I have also done the regressions by using the nominal values; this did not change the fundamental results.

46 Unfortunately, there is no data for income at school district's level. So I used LTP as an approximation. A simple regression shows that at the county level, the LTP is positively correlated with PIPC (with a coef=0.165 and P-value 0)

47 Some examples: *Comprehensive Health Education* and *English Language Proficiency* are state grants. *Child Care Grant* and *Coordinated School Health Programs* are federal grants.

Table 4.2: 2012 Revenue Sources for public education, Colorado

RevSrc	2012 Total (million)	2012 Per Pupil	% of Total Rev
Local Tax Contribution	3,440	4,255.07	38.53%
State Tax Contribution	3,330	4,120.01	37.31%
Federal Grants	736	911.27	8.25%
State Grants	450	557.16	5.05%
Private Partnership Grants	72	89.1	0.81%
Other Discretionary Income	702	868.64	7.87%
Long Term Debt Proceeds	195	241.91	2.19%

Source: Colorado Department of Education

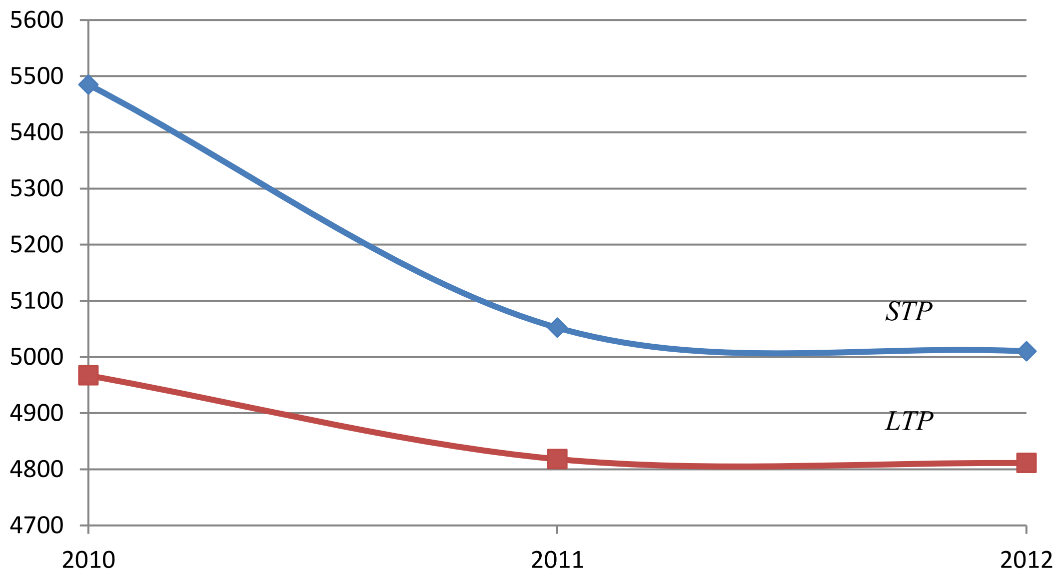

Figure 4.1: mean values of *STP* and *LTP* at district level, year 2010 to 2012

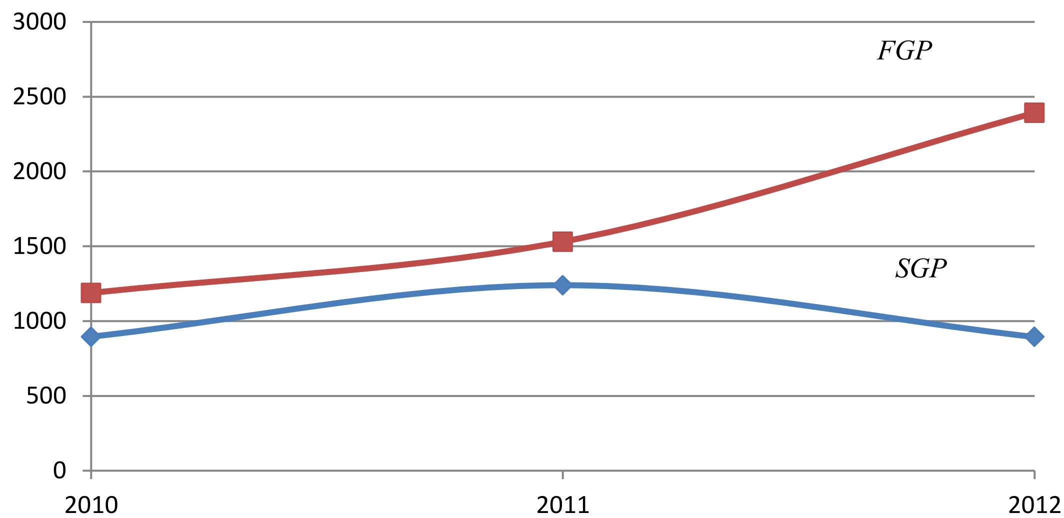

Figure 4.2: mean values of *FGP* and *SGP* at district level, year 2010 to 2012

Table 4.2 reports the revenue sources for public schools in the state of Colorado in the year 2012. The two major sources, local and state taxes counted as much as 75% of the total revenues. Note that at the state level, state tax contribution (37.31%) is almost as equal as the local tax contribution (38.53%). Federal grants and state grants counted as more than 13% of the total revenues and federal grants are nearly 400 USD greater than that of state grants in per pupil revenue. By comparing the magnitudes of tax revenue and other resources we may argue that if the state government in Colorado is trying to equalize the per student spending across districts, state funding should play the major role.

Figure 4.1 and 4.2 illustrate how the mean values of our major variables change over time. As can be seen from these above two figures, LTP and STP decreased significantly from the year 2010 to 2011, SGP increased in the three years and FGP is relatively stable. The fall in STP might be caused by a budget cut when the state government faced the depression. Note that the variables in both figures are only the mean values of the observations. They may not precisely reflect the mean value change at the state level.

The fundamental regression model is:

$$STP_{it} = f(LTP_{it}, FGP_{it}, SGP_{it}) + dummy \ \ (Year) + e_{it}$$

where i indexes time and t indexes the school district. All money terms are inflation adjusted.

According to the analysis, if the state government is playing a redistributive role, the coefficient for LTP should be negative and significant. FGP and SGP are also included to examine whether the federal and state grants are used as redistributive tools.

At the end of section, I list all the assumptions used in my regression models, which are also used in Chapter 2 and Chapter 3:

- Households can only send their children to schools in their own districts.
- Households stratify themselves into districts based on their income. If district $d_{\tau+1}$' mean income is greater than that of district d_{τ}, then the poorest household in $d_{\tau+1}$ has an income no less than the richest household' income in district d_{τ}.
- There is no migration cost so households can freely move from one district to another.

4.5 Models and Results

4.5.1 Model One

The first question needs to be answered from our regression models is whether the Colorado state government plays a redistributive role in financing the state public education. In other words, we would like to know how the state funding (STP) received by school districts is correlated with local wealth condition (LTP). If there is a significantly negative relationship between the two variables, the state government plays a redistributive role. Otherwise, Colorado state government is not reducing the education expenditure disparity across school districts.

We need a basic panel regression model to test the relationship. Dummy variable $Year$ is included to see whether the state financing differs in different years. FGP and SGP are also in the model to test if these two grands are used as redistributive tools. This basic model is presented as model one:

$$STP_{it} = a_0 + a_1 LTP_{it} + a_2 FGP_{it} + a_3 SGP_{it} + \gamma_i + \delta_t + e_{it}$$ [48]

In Table 4.3, the coefficient for LTP is negative and significant. This provides strong evidence that the state government is redistributing education resources from the rich districts toward the poor districts.

Table 4.3: Model one, the basic panel regression

STP	Coef	Robust Std.Err	t	$P > \|t\|$
LTP	$-.4260$	$.0741$	-5.75	0.000
FGP	$-.0013$	$.0483$	-0.03	0.978
SGP	$.0047$	$.0026$	1.77	0.079
Year_2011	-736	43.96	-16.75	0.000
Year_2012	-871	49.74	-17.52	0.000
Constant	7977	380.7	20.95	0.000

Coefficients for both ***FGP*** and ***SGP*** are insignificant in the regression. This suggests that federal and state grants are not used as redistributive tools. The coefficients for the year 2011 and 2012 are -736 and -871, respectively. Compared to the year 2010, the average state funding received by the school districts decreased by 736 USD in 2011; the number for 2012 is 871 USD. These decreases in STP reflect the fall in the state budget on public education.

From model one, we can see that the $LTP - STP$ curve is downward sloping. Now I want to further examine the change of the slope as the LTP changes. For the sake of convenience, I define the *redistribution rate* as the amount the state government reduces in STP when a district's wealth

[48] I use fixed year and district effects for all regression models.

increases by one additional dollar[49]. More specifically, the redistribution rate equals the absolute value of the $LTP - STP$ slope.

4.5.2 Model Two

Since we have already determined that the Colorado state government is playing a redistributive role in financing the public education, now we turn to the second question posted at the beginning of this paper: whether the currently used state education financing policy satisfies the necessary condition of Pareto-improvement. In order to test the second question, I introduce model two to examine the change of the slope (redistribution rate) as local wealth changes:

$$STP_{it} = b_0 + b_1 rich \cdot LTP_{it} + b_2 poor \cdot LTP_{it} + b_3 FGP_{it} + b_4 SGP_{it} + \eta_i + \zeta_t + \varepsilon_{it}$$

Here, I create two dummies: $rich$ and $poor$. If one district's LTPs are above the mean values for at least two years, then it is in the rich group and has $rich = 1$ and $poor = 0$; otherwise, it is in the poor group with $rich = 0$ and $poor = 1$.

Table 4.4: Model two, panel regression with two-district-group

STP	Coef	Robust Std.Err	t	$P > \|t\|$
$poor \cdot LTP$	$-.3352$	$.1440$	-2.33	0.021
$rich \cdot LTP$	$-.4546$	$.0802$	-5.67	0.000
FGP	$-.0067$	$.0485$	-0.14	0.890
SGP	$.0035$	$.0032$	1.10	0.275
Year_2011	-733	43.35	-16.93	0.000
Year_2012	-875	46.37	-18.87	0.000
Constant	7919	378.6	20.92	0.000

49 For the poor districts, this can be also interpreted as the amount received from the state government as their wealth decreases by one additional dollar.

From Table 4.4, we can see that both b_1 and b_2 are negative, which is consistent with the result in model one. However, F-test (F value equals 0.4626) shows that there is no statistically significant difference between these two coefficients, thus there is no evidence of a nonlinearity under rich and poor two-group-division. Statistically, $b_1 = b_2$ also indicates that the redistribution rates in the $poor$ districts and the $rich$ districts can be considered as equal, so when local wealth increases by one dollar, the fall in state funding for districts from both $poor$ and $rich$ groups are almost the same.

4.5.3 Model Three

In the previous model, I try to find out the existence of nonlinearity among school districts with different wealth conditions and the change of redistribution rate when state government is funding local public education. Although results in model two only provide evidence of linearity, the nonlinear relationship between STP and LTP may still exist. This is because how to divide school districts into different groups based on local wealth condition is rather arbitrary, and an arbitrary grouping is not very likely to provide us all the details in the relationship between STP and LTP. So I introduce model three, a polynomial model such that a global view of the redistribution rate can be provided.

In this model, I introduce the quadratic term $LTP^2 * 10^{-4}$ and the cubic term $LTP^3 * 10^{-9}$[50], the relationship between STP and LTP can be presented on a smooth curve, so we can examine the change in the redistribution rate as a whole. 10^{-4} and 10^{-9} are used to keep the coefficients in the same magnitude.

$$STP_{it} = d_0 + d_1 LTP_{it} + d_2 LTP_{it}^2 * 10^{-4} + d_3 LTP_{it}^3 * 10^{-9} + d_4 FGP_{it} + d_5 SGP_{it} + \theta_i + \vartheta_t + \sigma_{it}$$

[50] LTP^4 is not significant so I do not include in the model.

F-tests illustrate a strong evidence of nonlinearity since both d_2 and d_3 are significantly different from zero (with F values of 0.0003 and 0.0004 respectively). A basic graph of the polynomial function is provided as the following and we can see that the redistribution rate (absolute value of the slope) increases and then decreases as LTP increases.

Table 4.5: Model three, a polynomial regression

STP	Coef	Robust Std.Err	t	$P > \|t\|$
LTP	.1357	.1944	0.70	0.486
LTP^2	−.8475	.2306	−3.67	0.000
LTP^3	3.154	.8656	3.64	0.000
FGP	.0058	.0442	0.13	0.895
SGP	.0035	.0030	1.17	0.242
Year_2011	−739	45.15	−16.37	0.000
Year_2012	−881	50.91	−17.32	0.000
Constant	7171	543.8	13.19	0.000

Why is the redistribution rate of key importance? Because it determines whether the current policy is potentially Pareto-improving or not. When the redistribution rate is greater than 1, $\Delta LTP < \Delta STP$ as we move along the curve. For the residents, they will move to a less wealthy district because the increase in STP outweighs the decrease in LTP and the necessary condition of Pareto-improvement is satisfied. On the other hand, when redistribution rate is less than one, residents will find wealthier district more attractive and the currently used policy is not Pareto-improving.

Tests of coefficients from model one to model two show that the redistribution rate is significantly less than 1. The graph of model three illustrates that the redistribution rate is maximized when a district is from middle-income district group[51], and this motivates the introduction of model four.

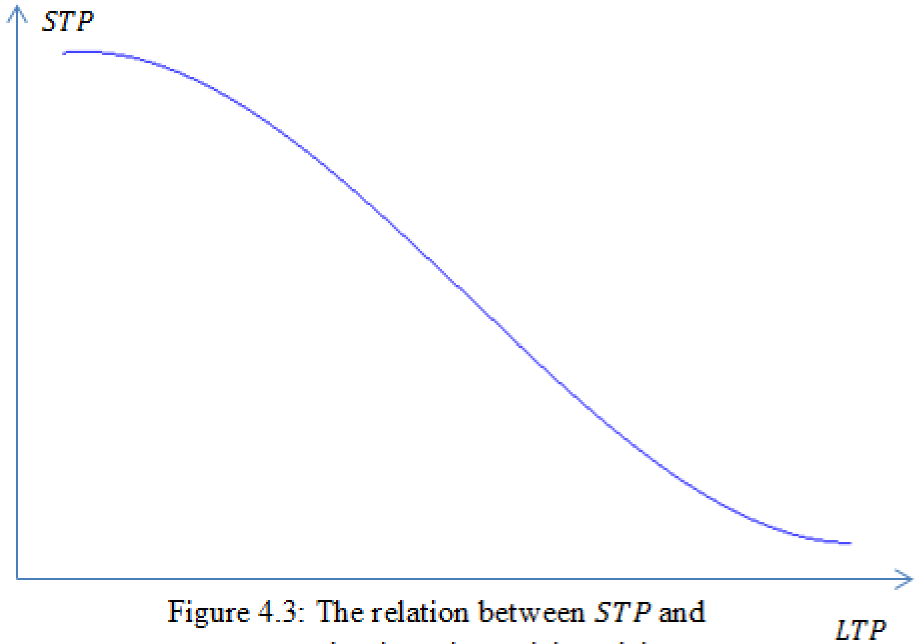

Figure 4.3: The relation between STP and LTP under the polynomial model

4.5.4 Model Four

In this model, all households are divided into three groups based on the results of model three. Model three presents a "flat-steep-flat" curve between LTP and STP as local wealth increases, thus I divide all school districts into three groups: low, $middle$ and $high$. If $8000 < LTP_{it} < 12000$ for more than two of the three years, one district is in the $middle$ group, if $LTP_{it} > 12000$ for two or more years, a district is in the $high$ group, and the rest districts are in the low group. In model four, I want to examine the coefficients of the three groups, and model four is presented as the following:

$$STP_{it} = c_0 + c_1 low \cdot LTP_{it} + c_2 middle \cdot LTP_{it} + c_3 high \cdot LTP_{it} + c_4 FGP_{it} + c_5 SGP_{it} + \kappa_i + \lambda_t + \epsilon_{it}$$

[51] By the first and second order condition of the polynomial function, $\partial^2 STP/\partial LTP^2$ implies that $LTP^* = 10569$. Given $LTP \in [0,17880]$, it is somewhere close to the middle.

Table 4.6: Model four, panel regression with three-district-group

STP	Coef	Robust Std.Err	t	$P > \|t\|$
$low \cdot LTP$	$-.4514$	$.0961$	-4.70	0.000
$middle \cdot LTP$	$-.8404$	$.0983$	-8.55	0.000
$high \cdot LTP$	$-.2711$	$.1445$	-2.37	0.019
FGP	$.0050$	$.0444$	0.11	0.911
SGP	$.0058$	$.0029$	1.97	0.050
Year_2011	-739	44.05	-16.78	0.000
Year_2012	-879	47.47	-18.52	0.000
Constant	8194	358.98	22.83	0.000

Tests of coefficients show that $Coef_{low} > Coef_{middle}$, $Coef_{high} > Coef_{middle}$ and $Coef_{low} = Coef_{high}$, with all of these results being statistically significant[52]. The test of $Coef_{middle} = -1$ provides an $F-value = 0.1063$ and the test of $Coef_{middle} \leq -1$ provides a $P-value = 0.0532$, indicating that we can only accept the hypothesis that the redistribution rate for $middle$ group districts is greater than 1 at any significance level below 5.32%[53]. Both $Coef_{low}$ and $Coef_{high}$ are statistically greater than -1[54]. Thus we can conclude that the redistribution rates for low and high wealth districts are strictly less than 1, and for most of the middle districts, the redistribution rate is less than 1.

Figure 4.1 shows the results of model four, the curve follows a "flat-steep-flat pattern" which is consistent with the results in model three.

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[52] The hypothesis $c_1 > c_2$ provides a P-value equals 0.998, $c_3 > c_2$ provides a P-value equals 0.999 and the F-value for $c_1 = c_3$ is 0.22.

[53] So we cannot reject the hypothesis at the 5% level, but we can at the 10% level.

[54] The hypothesis $c_1 \geq -1$ and $c_3 \geq -1$ result in P-values equal 0.

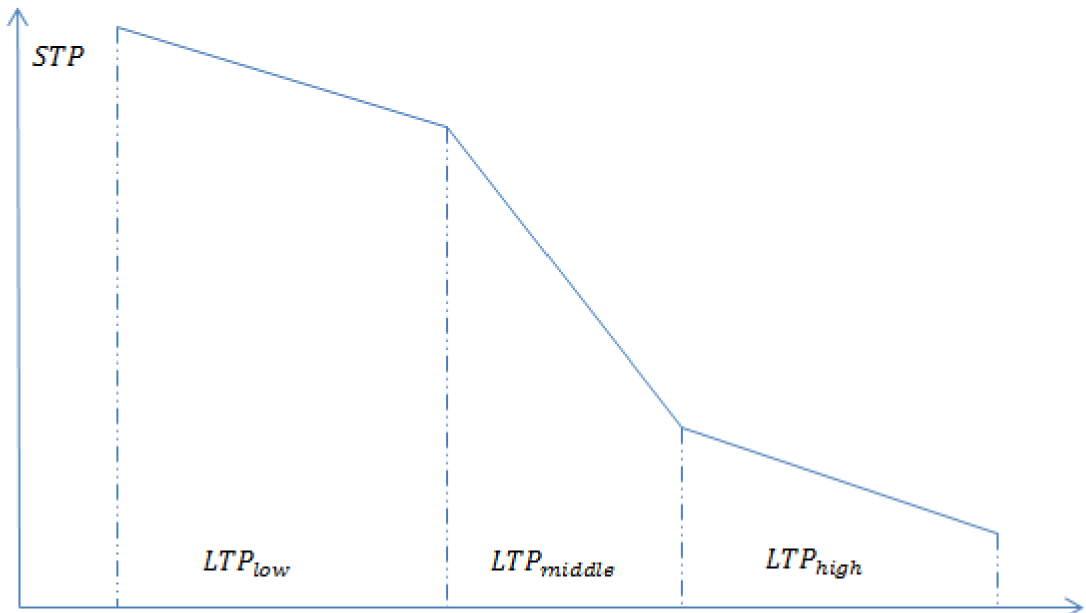

Figure 4.4: The relation between STP and LTP when LTP is divided into three groups

4.6 Policy Analysis

The slope of STP under the current policy follows a "flat-steep-flat" pattern, indicating that the rich and poor school districts lose less state funding when local wealth increases than the districts in the middle. More importantly, the redistribution rates for these two groups are definitely less than one according to the tests, while the redistribution rate for the middle group districts is not so clear. However, based on the test results in model four, it is reasonable for us to believe that $|Coef_{middle}| < 1$ for most of the districts with middle wealth condition.

According to the definition of redistribution rate, when this rate is less than one, the slope (absolute value) of the $LTP - STP$ curve is also less than one. As shown in the left part of Figure 4.5, it illustrates cases for both low-income districts and high-income districts: when we are moving downward the curve, $\Delta LTP > \Delta STP$. Thus for residents from a relatively poorer district, they will always find that it is beneficial to move to wealthier districts because the increase in local tax spending per pupil outweighs the loss in state average expenditures. The net effect of migration "from less wealthy to wealthy" and residents will follow this migration pattern, which is not consistent with the necessary condition of the Pareto-improvement (a migration pattern "from wealthy to less wealthy").

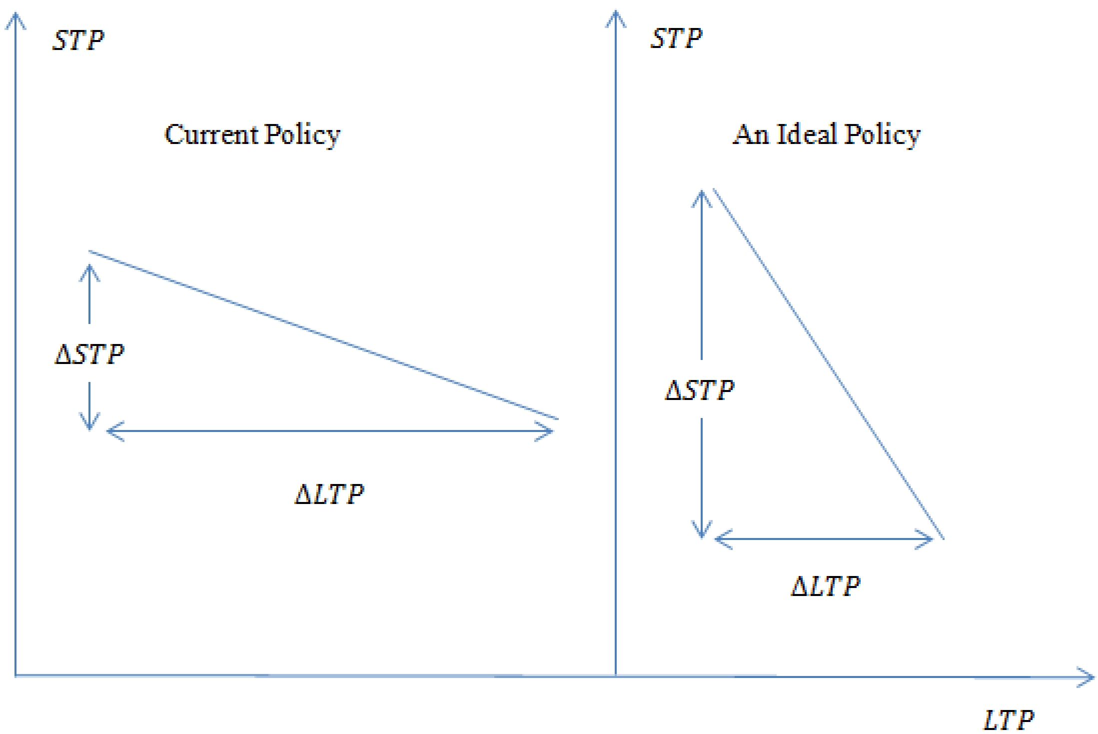

Figure 4.5: Current Policy *VS* Ideal Policy

An ideal policy is shown on the right of Figure 4.5 when the redistribution rate is greater than one. Under this ideal policy, $\Delta LTP < \Delta STP$ and residents will find it beneficial to move to a relatively less wealthy district and the policy is potentially Pareto-improving.

The policy for the middle group districts is ambiguous because the redistribution rate is not significantly less than one at all significance level. For those districts with a distribution rate less or equal than one[55], the state government needs to adjust the state funding amount to alternate the slope, for those very few districts (it is also likely thus kind of districts do not exist) with redistribution rate greater than one, the state government should continue using the current policy.

[55] If redistribution rate equals one, the residents are indifferent between moving and stay, we can just simply assume they will not move in this case.

In Chapter 3, we have discussed the role that migration plays in education financing system when the state government tries to redistribute education resources between communities. If migration is to be taken into consideration, a basic regression model should be written as:

$$STP_{it} = f(LTP_{it}, FGP_{it}, SGP_{it}, Mig_{it}) + e_{it}$$

Unfortunately, we do not have any school district migration data, so the above model cannot be run at school district level. Although the regression can be run at the county level (migration data are available at the county level), the county regression model imposes a bold assumption that LTP is the same for every district within one county[56]. More importantly, this model omits the migration among school districts within the same county. Since there is no way to embrace the migration into the model, I decide to exclude migration in my regressions.

To conclude, the currently used redistributive policy in Colorado is hardly Pareto-improving. Although it is still possible that in a few middle wealth districts[57], boundary residents will move to the relatively poorer districts, the majority of districts with high and low local wealth condition are experiencing a redistribution rate less than one. When residents find wealthier districts more attractive since the increase in local expenditures outweighs the decrease in state funding, the "wealthy to less wealthy" migration chain breaks. We may very likely have the case that in many districts, the none-boundary residents have to face both decreasing mean income and total expenditures per student. As such, they will become worse off.

4.7 Conclusion and Policy Suggestion

Two major results can be drawn from the empirical study of the state financing in Colorado:

1. The Colorado state government is playing a redistributive role in financing the public education. It collects tax revenues from all school districts and spends larger amount per pupil on poor districts.

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[56] Even within one county, LTP varies greatly among different districts. Estes Park, Poudre and Thompson are all in Larimer county, however, local LTP in 2010 for these three districts are 9932, 4898 and 4056 respectively.

[57] In model four, the middle group only has 42 observations, and the total observations are 537.

2. The redistribution rate under the current policy increases and then decreases as district's LTP increases. For all poor, rich and most of the middle districts, the redistribution rate is less than zero, resulting in a migration pattern from less wealthy districts to relatively wealthier districts, which does not satisfy the necessary condition for Pareto-improvement.

The first result suggests the state government is doing what it is supposed to do in general. However, the currently used policy is not potentially Pareto-improving. Theoretically speaking, the best policy for the state government is to redistribute only between the richest and the poorest districts. But it is unrealistic for a few richest districts pay taxes and another few poorest districts receive state education fund.

A much more feasible way is to change the redistribution rate under the current policy: the state government should take more marginal tax revenues from the rich districts as local wealth increases and increase the marginal subsidy for the poor districts as local wealth falls. To be more specific, under the current policy, the redistribution rates for the rich group and the poor group are 0.4 and for most of the middle districts, the rate is 0.8. This indicates that as LTP increases by one dollar, the state government reduces the average state funding received by the school districts in the rich and poor districts by 40 cents, the number for most of the middle districts is 80 cents. In the ideal policy, the redistribution rate for all districts needs to be less than 1 in order to ensure the policy is potentially Pareto-improving. If the state government reduces 1.5 dollars in average state funding as LTP increased by 1 dollar (or in another word, increases state funding by 1.5 dollars as local wealth decreased by 1 dollar), households will be attracted to move to relatively poor districts, thus a potential Pareto-improvement is achieved.

CHAPTER 5

CONCLUSION

This dissertation examines the financing system of public education from both theoretical and empirical perspectives. In the theoretical sections, models are built to study the provision of local public education and examine whether a Pareto-improving reform exists under current financing system. In the empirical chapter, I focus on the state financing policy in Colorado, four regression models are employed to examine: ①whether the state government is reducing disparity in education expenditures across school districts and ② whether the current state financing policy is potentially Pareto-improving.

Fernandez and Rogerson's work (1996, 1997 and 1998) cannot be omitted when investigating the financing system of public education from the theoretical perspective. In their 1996 paper, Fernandez and Rogerson built a model simple enough yet captured all major factors in examining the provision of local public education: heterogeneous individuals and communities, taxation, migration and local public education. Fernandez and Rogerson (1996) discussed the characteristics of the model under a two-community and three-income-group case and presented some potentially Pareto-improving reforms. By adding additional assumptions on population distribution and individuals' preferences, I analytically show that the reform "to redistribute a fraction of education expenditures away from the rich community toward the poor community" is Pareto-improving. This reform still works when a perfectly competitive housing market is introduced into the model and the local public education is financed on a property tax instead of an income tax. At the end of my second chapter, I also derive the necessary condition of Pareto-improvement when there are a large number of communities.

In Chapter 3, I examine the provision of local public education under a more complicated and more realistic model. A general housing market with an upward sloping supply curve is used and local public education is funded by a property tax rate which is proportional to the housing price.

Numerical methods are employed in this chapter and the simulation model partly follows Fernandez and Rogerson (1998). Results show that the reform posed in Chapter 2 can still be Pareto-improving, the redistributive fraction chosen by the state government is the key factor that determines whether the Pareto-improving policy works or not. If the redistributive fraction is too small, the utility loss in the poor community cannot be reversed, if the fraction is too large, households in the rich community will become worse off because per student spending in the rich community falls below the desired level. I also use the simulation model to examine the welfare change of major counties in Colorado, among counties with positive county-to-county migration rate, Weld and Yuma are better off, and most of the counties with negative county-to-county migration rate are better off.

In the empirical chapter, the state financing system on public education is investigated in Colorado. Regression results show that the state government is trying to reduce the disparity in per student spending across school districts. Generally speaking, rich school districts tend to receive less state support and poor districts tend to receive more education funding from state government. However, the current policy is not potentially Pareto-improving, when local wealth increases, the increase in local support is greater than the decrease in state support and the total effect on per student spending is positive. Thus, households living in the less wealthy districts want to move to wealthier ones. Suggestions are made to turn the current policy into a potentially Pareto-improving one.

There are four things need to be paid attention to about the regressions and the theoretical model. (1) In Colorado as well as in other states, the public school choice program exists and families do not have to send their children to schools in the districts where they live. According to *National Center of Education Statistics*, in the year 2007, among all households choose public education for their children, 82.5% of the parents send their children to assigned public schools. So for all chapters, it is assumed that students can only attend schools in their own districts. However, from the economic perspective, households who join the school choice program are the "free riders" in the education financing system because they enjoy better public education than they deserve (we can hardly imagine parents send their children to districts with schools worse than their own). In order to drive "free riders" back to their own districts, state government can increase distribution rate and make free riders' own districts more attractive to themselves, so in Chapter 4, the absolute value of the coefficient in an ideal policy may be biased down.

(2) The theory also predicts a long run equilibrium for the dynamic model. The rich district does not want to mimic the poor one because state government observes mean incomes and directly takes part of the tax revenue from the rich district and uses it to subsidize the poor district. Individuals with higher incomes also prefer higher expenditures on public education, so they will not vote for a low tax rate. In reality, the state government uses indirectly redistributive policy. It collects tax revenue from all school districts and returns it in the form of state education funding. The data does not provide any evidence showing that the rich districts are mimicking the poor ones, this may due to government regulation or other factors outside the model[58].

(3) The definition of "the quality of public education". In my dissertation and some literature (e.g., Fernandez and Rogerson, 1996, 1997 and 1998 and Calabrese et al., 2006), the quality of public education is assumed to be equal to the spending per student. In many other research papers (e.g., Channa and Faguet, 2016 and Goldhaber et al., 2011), test scores are used as the quality of public education. To certain extent, the test score is a better method than the per student spending in measuring the quality of public education since it is one of the most important outcomes for children attending schools. However, test scores cannot be used as the quality of public education in my dissertation, this is because I investigate how the reform on financing system would affect the locally provided public education, the per student spending is directly related to the financing system.

(4) The migration of students/households within districts/communities. In my theoretical model, housing price, tax rate and per student spending within one community is the same for

[58] If the rich districts mimic the poor district (e.g., voting for a low tax rate), the provision of public facilities other than public education will decrease, resulting in welfare loss for the rich people. In addition, poor people may move into the rich districts and crime goes up (according to *Bureau of Justice Statistics 2014*, rate of violent victimization in poor group is more than twice as much as that in high-income group), which leads to negative externalities for all rich households.

every household, so the migration of household (from one neighborhood to another) or student (from one school to another) within one district/community has no impact on model results. In the empirical analysis, we do not have data for per student expenditure or enrollment at school level in Colorado. Thus, I do not include the discussion of within-district migration in my dissertation.

Future research may include topics in both theoretical and empirical aspects. In the theoretical model, an income function can be used to relate the quality of public education with future income, by doing so we can examine the reform policy in an economy with infinite periods. From the empirical perspective, we can include students' academic scores in our regression model and examine whether the state financing policy reduces disparity in academic performances across districts.

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APPENDICES

A.1 Redistributive Reforms: Colorado, year 1999, 2005

Table A.1: U_constant redistribution, Colorado, year 2005

Year = 2005	Before the Reform	After the Reform
p_1	1.603	1.534
t_1	0.172	0.151
q_1	2.876	2.119
p_2	1.169	1.338
t_2	0.327	0.174
q_2	1.3207	1.4612
U_1	0.0686	0.0686
U_2	−0.009	−0.0083
U_3	−0.2072	−0.2068

Table A.2: U_constant redistribution, Colorado, year 1999

Year = 1999	Before the Reform	After the Reform
p_1	1.568	1.502
t_1	0.173	0.146
q_1	2.6192	1.8715
p_2	1.141	1.307
t_2	0.318	0.167
q_2	1.1513	1.2660
U_1	0.0259	0.0259
U_2	−0.0563	−0.0556
U_3	−0.2729	−0.2726

Table A.3: E_constant redistribution, Colorado, year 2005

Year = 2005	Before the Reform	After the Reform
p_1	1.603	1.534
t_1	0.172	0.151
q_1	2.876	2.876
p_2	1.169	1.324
t_2	0.327	0.251
q_2	1.3207	1.4465
U_1	0.0686	0.0694
U_2	−0.009	−0.0095
U_3	−0.2072	−0.2091

Table A.4: E_constant redistribution, Colorado, year 1999

Year = 1999	Before the Reform	After the Reform
p_1	1.568	1.502
t_1	0.173	0.146
q_1	2.6192	2.6192
p_2	1.141	1.292
t_2	0.318	0.252
q_2	1.1513	1.2533
U_1	0.0259	0.0269
U_2	−0.0563	−0.057
U_3	−0.2729	−0.2754

A.2 Matlab Codes

HEAD TAX

The Matlab codes are written to solve the housing price (p) and tax rate (t) for each community given by equations:

$$\begin{cases} u_c h = v'\bar{y}[g(\pi) + tp \cdot g'(\pi)] \\ \quad\quad ap^b = Ng(\pi)\bar{y} \end{cases}$$

The scripts below are used to solve the case for the rich community, year 2008. I first write the terms in the equations in eight functions and then plug into the test.m script to find out the solution.

```
function result=g(t,p)
pi=p*(1+t);
ac=0.936;aq=0.081;del=-3.9;alp=-0.6;B=8;N=0.5;mu=120;y=150;
result=((pi*ac/(1-ac))^(1/(1-alp))+pi)^(-1);
```

```
function result=gd(t,p)
pi=p*(1+t);
ac=0.936;aq=0.081;del=-3.9;alp=-0.6;B=8;N=0.5;mu=120;y=150;
result=-g(t,p)^2*((ac/(1-ac))^(1/(1-alp))/(1-alp)*pi^(alp/(1-alp))+1);
```

```
function result=h(t,p)
ac=0.936;aq=0.081;del=-3.9;alp=-0.6;B=8;N=0.5;mu=120;y=150;
result=y*g(t,p);
```

```
function result=pt(t,p)
ac=0.936;aq=0.081;del=-3.9;alp=-0.6;B=8;N=0.5;mu=120;y=150;
result=N*mu*p*gd(t,p)/(3.5*p^(2.5)-N*mu*(1+t)*gd(t,p));
```

```
function result=q(t,p)
ac=0.936;aq=0.081;del=-3.9;alp=-0.6;B=8;N=0.5;mu=120;y=150;
result=t*p*g(t,p)*mu;
```

```
function result=qt(t,p)
pit=pt(t,p)*(1+t)+p;
ac=0.936;aq=0.081;del=-3.9;alp=-0.6;B=8;N=0.5;mu=120;y=150;
result=p*g(t,p)*mu+t*g(t,p)*mu*pt(t,p)+t*p*mu*pit*gd(t,p);


function result=uc(t,p)
pi=p*(1+t);
ac=0.936;aq=0.081;del=-3.9;alp=-0.6;B=8;N=0.5;mu=120;y=150;
result=ac*y^(alp-1)*((pi*ac/(1-ac))^(1/(1-alp))/((pi*ac/(1-ac))^(1/(1-
alp))+pi))^(alp-1);


function result=vd(t,p)
ac=0.936;aq=0.081;del=-3.9;alp=-0.6;B=8;N=0.5;mu=120;y=150;
result=aq*B*(1+q(t,p))^(del-1);


% solve the equations, test.m %
clear
clc
ac=0.936;aq=0.081;del=-3.9;alp=-0.6;B=8;N=0.5;mu=120;y=150;
find=0;
A=cell(100,1)
index=1;
p=1.5; t=0.14;
for i=1:100
    p=1.5;
    for j=1:200
    [p,t]
    R1=p^(3.5)-N*g(t,p)*mu;
    R2=uc(t,p)*h(t,p)*(pt(t,p)*(1+t)+p)-vd(t,p)*qt(t,p);
    if (abs(R1)<0.0008 && abs(R2)<0.0008)
       A{index}=[p,t];
       index=index+1
    end

    p=p+0.001;
    end
```

```
   % if(find==1)
     t=t+0.001;
    %end
end
[p,t]


% This file is created to calculate the provision of public education,
% and the utilities for all income groups living in community one before
% the reform %
clear

% Enter the parameters %
ac=0.936;aq=0.081;del=-3.9;alp=-0.6;B=8;N=0.5;mu=120;y0=3.01;

% Enter the simulation results %
p=1.666;t=0.168;

% Enter the function%
pi=p*(1+t);
g=((pi*ac/(1-ac))^(1/(1-alp))+pi)^(-1);

% Calculate the provision of public education%
q=t*p*g*mu;

% Enter the income for each group%
y1=150;y2=60;

% Housing demand for each group%
h1=y1*g;
h2=y2*g;


% Private consumption for each group%
c1=y1-pi*h1;
c2=y2-pi*h2;


% Utility from receiving education (this is the same for every group) %
vq=aq*(y0+B*(1+q)^del/del);
```

```
% Utility from housing and private consumption %
u1=(ac*c1^alp+(1-ac)*h1^alp)/alp;
u2=(ac*c2^alp+(1-ac)*h2^alp)/alp;


% Total utility for each group %
U1=u1+vq;
U2=u2+vq;
```