Atmospheric Structure and Clean-Air Turbulence

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E. R. Reiter and A. Burns

Abstract

Measurements of clear-air turbulence spectra conducted by a Canberra aircraft over Australia between August and October 1963 reveal the existence of a wavelength region from approximately 2000 to 4000 ft, in which the atmosphere receives turbulent energy mainly in the w-component of motion. It is suggested that this energy stems from gravitational shearing waves which break up into turbulent eddies below a critical wave length. The energies of these turbulent eddies seem to be well represented by a proportionality to $\omega^{-5/3}$, characteristic of the inertial subrange of turbulence.

Introduction:

Clear-air turbulence (CAT) still remains a major hazard of modern aviation, which may result in severe damage or even loss of aircraft (Reiter, 1963a, 1964a). Structural fatigue and passenger discomfort are factors which have to be taken into account even in less dramatic encounters with CAT. Our knowledge of this phenomenon has been summarized by the author and by others in previous publications (Reiter, 1960, 1961, 1962a; 1963b, c, 1964b, c, d; Reiter and Hayman, 1962; Reiter and Nania, 1964; Endlich and McLean, 1964; Hildreth, et al., 1963; Panofsky and McLean, 1964; Pchelko, 1962; Pinus and Shmeter, 1962; Clodman et al., 1960).

The present study is aimed to highlight certain implications of CAT with respect to atmospheric structural characteristics that might eventually lend themselves to exploration by radio-wave propagation methods.

Energy Spectra of CAT

It has been shown by Kolmogorov (1941), Obukhov (1941), Heisenberg (1948) and by others (see e. g. Batchelor (1959), and Lumley and Panofsky (1964)) that the energy spectra of turbulence in the inertial subrange may be expressed as

$$E(\omega) = \alpha \epsilon^{2/3} \omega^{-5/3} \tag{1}$$

$E(\omega)$ is the (frequency-dependent) kinetic energy of turbulent motion in the frequency range ω to $\omega + d\omega$, α is a universal constant, ϵ is the rate of dissipation of energy.

Within the inertial subrange the energy spectrum is independent of the kinematic viscosity. No production or viscous dissipation of turbulent energy is assumed to take place in this range. Turbulent energy is simply transferred to smaller and smaller eddies.

The validity of the "$^{-5}/3$ law" has been well substantiated in low-level turbulence observations. Recent investigations of turbulence measured by aircraft indicated, that such an "inertial subrange" of turbulence also exists in the free atmosphere, in clouds as well as in clear air.

Typical power spectra of turbulence in thunderstorms, in cumulus clouds, and in clear air at low levels were obtained by Rhyne and Steiner, 1962. MacCready (1962, 1964) reports on spectra measured by sailplane. Another set of aircraft turbulence data has been presented by Shur (1962) for well and weakly defined jet streams, as well as for CAT over mountains, in cumulus and in cirrus clouds. Shur's power spectra pertain to vertical velocity components only, which were derived in a manner somewhat different from that of the above investigators. He makes use of aircraft accelerometer readings, deriving the spectra from the relationship

$$E_o(\omega) = E_i(\omega) . \left| T(\omega) \right|^2 \tag{2}$$

where $E_o(\omega)$ is the measured "output" spectrum function, e. g. the gust loads on the aircraft as determined from the accelerometer records; $E_i(\omega)$ is the "input" spectrum function, i. e. the spectral density of atmospheric (vertical) gusts causing the observed accelerations of the airplane; $T(\omega)$ is the absolute value of the frequency response function of the aircraft, describing the rigid and elastic oscillation modes of the aircraft in response to sinusoidal gusts of various frequency.

During Project "TOPCAT" an additional set of CAT data has been obtained over southern Australia during the period from July 21 to October 3, 1963. (Burns and Rider, 1965; Mizon, 1964; Radok, 1964; Reiter, 1964e; Spillane, 1964). Measurements were made by a Canberra aircraft flying at altitudes of 26,000 to 33,000 ft (9-11 km). The airplane carried on a nose boom a differential pressure probe which measured separately small pressure fluctuations caused by atmospheric gusts in the u, v and w component. (Coordinates are given with reference to the motion of the aircraft). In addition, acceleration, gyroscopic, and strain gauge measurements were available to allow a study of aircraft responses.

As described by Burns and Rider (1965) special care has been taken to eliminate contamination of the records by aircraft motions, such as the "Dutch roll". In spite of the restrictions inherent in the mathematical treatment of response characteristics, the atmospheric turbulence spectra obtained should, therefore, be considered quite reliable. Specifically, contamination of the records by the pilot's maneuvers was eliminated, thereby reducing a problem which Shur (1962) was faced with. This was done mainly by including correction for recorded aircraft motions, both pilot and gust induced. Furthermore, the pilot was instructed to keep control movements to a minimum.

Corrective handling of the controls by the pilot would mainly affect the low-frequency end of the observed spectra. That any such effects were negligible is shown in Fig. 1 which contains spectra of the u, v and w component of Flight 45, Run H, once computed for the entire run ($3\ ^1/2$ minutes), once for the last half of the run ($1\ ^2/3$ minutes). There is no significant difference between the two groups of spectra, especially not in their long-wave end. Furthermore, from this diagram it may be seen that the spectrum analyses do not suffer significantly by the relative shortness of the sample.

Vertical incremental accelerations in the CAT patches for which power spectra were calculated were of the order of 0.5 to 0.8 g. CAT was estimated to be moderate to severe. Table I (Burns and Rider, 1965) contains information on the research flights evaluated so far. Meteorological conditions for these flights are summarized in Table II.

As may be seen from Table I, the standard deviations of all three components of turbulence fluctuations in the wave-length region of 70 to 15,000 ft showed some variation. Nevertheless, the spectra reproduced in Figs. 2 to 4 have not been standardized because they were all characteristic to CAT near the "moderate" level. An exception is the set of data obtained at low levels which revealed considerably less turbulence energy, especially in the w-component, than the rest of the spectra.

Judging from subjective experience, CAT seems to consist mostly of "bumps" in a frequency range of more than one "bump per second". The airspeeds given in Table I, thus, would indicate CAT to occur in the wave-length range of <1000 ft. In agreement with this, the spectra, especially in the w-component (Fig. 4), show remarkable similarity at wave-lengths <1000 ft. An exception, again, is the spectrum obtained at 300 ft pressure altitude (Run 44). At wave lengths larger than ca. 2000 ft individual spectra of the vertical velocity component vary considerably for individual runs. The u and v spectra also show some dispersion for wave lengths > 2000 ft, although much less than evident in the w-component.

TABLE I (Burns and Rider, 1965)

SUMMARY OF TRAVERSES

Date	Flight and Run No.	Average Barometric Height-feet	Heading	Average Airspeed ft/sec, True	Duration of Run Seconds	Wind deg/ Kt	Mean Sq. Gust Velocity-ft/sec			Coefficient of Cross Correlation between w and u
							σ u*	σ v*	σ w*	
21/ 8/63	18.02 (Run B)	28,200	155°	734.1	37.5	270°/90	3.19	4.84	2.83	-0.188
21/ 8/63	18.04 (Run D)	29,100	260°	737.9	50	261°/90	5.09	6.25	3.39	0.289
4/ 9/63	27.06 (Run F)	26,690	280°	730.5	150	226°/94	2.95	3.45	2.78	0.069
12/ 9/63	33.05 (Run E)	32,040	255°	742.2	150	260°/90	4.02	4.00	2.60	-0.263
1/10/63	44.04 (Run D)	300	-	507.0	150	-	1.99	2.38	1.89	-0.022
3/10/63	45.08 (Run H)	28,450	086°	741.8	270	256°/43	4.36	2.93	2.39	-0.198
3/10/63	46.05 (Run E)	29,260	216°	744.4	100	278°/45	5.10	5.65	3.79	0.160

*Truncated values referring to wavelengths ranging from 70 ft. to 15,000 ft.

TABLE II:

Meteorological Conditions During Research Flights..

Date	Meteorological Conditions	CAT
(After Mizon, 1964)		
21.8.63	160 kt jet core between 28° and 30° S, westward of 140° E. Core at 250 mb. Strongest vertical shears near 300 mb.	Light to moderate CAT at 31° 40'S and 140° 27'E at 28,000 - 29,000 ft. Accelerations $\leq$ 0.5g.
(After Reiter, 1964e)		
4.9.63	Passage of jet stream and cold front over Flinders Range, east of Adelaide, should induce mountain-wave formation near sharply defined tropopause (27,000 ft).	Moderate to strong CAT along Flinders Range.
12.9.63	Strong jet core (balloon-measured winds ca.225 kt, maximum aircraft winds 133 kt) over Adelaide near 34,000 ft. Stable layer near 28,000 ft.	Thin cirrus observed north of Adelaide near 28,000 ft. Patch of moderate CAT first identified near 31° 48S, 137° 02E at 32,000 ft. Patch was marked repeatedly with smoke trail, and was followed for 45 minutes while drifting downstream with wind 80-100 miles. CAT 1000 ft. above and below main level was noticeably less. Pilot reported wave formation on smoke trail. CAT patch had to be abondoned for lack of fuel.
1.10.63	Low level comparison with DC-3 measurements near Wagga-Wagga.	See Table I.

TABLE II: (continued)

Date	Meteorological Conditions	CAT
3.10.63	Well developed confluence between two jet branches. Rapid turning of wind with height (196° to 260°) in layer 26,000 ft. to 31,000 ft. Generally light winds below jet-stream velocities in this region.	Extensive light to moderate CAT near shear line between 28,000 and 29,000 ft. One haze horizon observed at flight level, another one higher up. Some smoke puffs released by aircraft remained well rounded, some spread out into thin, nearly-horizontal, sheets indicating strongly shearing meso-structural layers.

For comparison, a line of " $^{-5}/3$ " slope has been entered. u- and v-spectra seem to fit this theoretical value of turbulent energy districution in the inertial subrange remarkably well. The fit, again, is best for wave lengths <2000 ft. On the average, the w-spectra for high-level CAT seem to be better approximated by a slope of $^{-4}/3$, especially for wave lengths between approximately 1000 and 200 ft.

The most conspicuous feature in these w-spectra seems to be a "hump" at wave length $\cong$ 2000 ft. Several spectra actually show a reversal of slope in the wave length range adjacent to, and larger than, 2000 ft. Such an irregularity is only weakly expressed in the u-spectra, and moderately well discernable in the v-spectra.

In Figs. 2 to 4 the reference lines of $^{-5}/3$ slope have been entered at the same energy levels. Since the position of the u-, v-, and w-spectra relative to this line are approximately the same to the right of the "hump" described above, i.e., for wave lengths < 2000 ft., we may conclude that the turbulence in this range is nearly isotropic.[1]

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[1] Of course, slight differences between longitudinal and traverse spectra may be expected even with isotropic turbulence (Pasquill, 1962, p. 7). These differences should be reduced, however, in Figs. 2 to 4, since u and v are defined with respect to aircraft motion and not with reference to the direction of mean flow. There are indications (Burns and Rider, 1965) that during head or tail-wind flights the u-spectra slightly exceeded the v-spectra in energy, while during cross-wind flights the opposite seemed to be true.

For wave lengths > 2000 the w-components show significant departures in turbulent energy from those of the u-and v-components, suggesting anisotropy of turbulence in this range. The inertial subrange, therefore, seems to be confined to waves < 2000 ft (Burns and Rider, 1965). This is in excellent agreement with Shur's (1962) findings, who also suggests 600 m as the upper border of the inertial subrange.

Interpretation of Data

Shur (1962) reports on a consistent irregularity which he found in CAT power spectra. For wave lengths < 600 m the spectra were well approximated by an exponent of -1.7 ($\cong -{}^{5}/3$). For wave lengths larger than 700 to 800 m the exponents ranged between -3 and -3.2. Shur attributes the steeper slope of the spectral curve at long wave lengths to an additional dissipation effect, caused by negative buoyant forces in a stable environment. His argument is, that turbulent energy will not only be dissipated at the rate ϵ, determined by the transfer of turbulent energy to smaller and smaller eddies, but also by work against Archimedean forces. Turbulent energy at wave length smaller than the ones of the source range - not yet established by Shur's measurements - will therefore decay more rapidly than indicated by the proportionality to $\omega^{-5/3}$.

Theoretical treatment of this "buoyant subrange" in the case of stable stratification has been offered by Bolgiano (1959, 1962) (see also Lumley and Panofsky, 1964). He arrives at the proportionality

$$E(\omega) \propto \omega^{-11/5}$$

Isotropy is not expected to prevail in this range of the turbulent spectrum.

Although Bolgiano's and Shur's exponents of ω in the "buoyant subrange" are not in agreement with each other, they both suggest a steeper slope of the spectrum curves at wave lengths larger than those of the inertial subrange. For comparison, both -3 and $^{-11}/5$ slopes have been entered into Figs. 2 to 4 of the spectra. In the w-components (Fig. 4) the long wave portion of the spectra to the left of the "hump" seem to have a preference for a "-3 slope" only in Run No. 27F. Runs No. 18B and 45H seem to line up with a $^{-11}/5$ slope in this region. Not too much significance should be attached to this statement, however, since there are too few data points available to establish the slope with confidence. The other runs actually seem to show a decrease of the slope of the spectrum curve in the w-component of long wave lengths.

The spectra of the v component seem to indicate slopes significantly steeper than $^{-5}/3$ at long wave lengths, whereas the u-spectra follow the $^{-5}/3$ slope, rather than a -3 or $^{-11}/5$ slope. No great significance should be attached to the apparent differences between u - and v - components, since they are given with reference to the flight direction rather than the mean-wind direction. Comparison with tower measurements at low-levels suggest that aircraft data slightly exaggerate the energies in the v-component at long waves.

It has been mentioned, that most of the w-component spectra show a significant "hump" near a wave length of 2000 ft. This means, that between waves of approximately 4000 and 2000 ft there is a source region of turbulent energy. In some cases, such as in run 18B, 27F, and 45H, 1.5 to 2 orders of magnitude more turbulent energy is made available to the inertial subrange (in which CAT is finally experienced) than would have been available, had the "buoyant subrange" to the left of the "hump" fed its energy at the normal dissipation rate, and without modification, into the "CAT range". Were it not for this additional energy source, the gust velocities in the CAT wave-length range should have been only $^1/10$ of the ones actually experienced. In other words, without the energy input from waves between 2000 and 4000 ft no CAT would have been encountered, and the flight conditions would even have been considerably smoother than during Run 44 at low-levels. Without direct proof from measurements, one might speculate that power spectra in "smooth air" should show such low energy levels.

A possible exception to the above reasoning is found in Run 18D, which shows considerable spectral intensity even at long waves, the spectral curve approximating a $^{-5}/3$ slope almost to its low-frequency end.

At least for those runs for which the "hump" is sharply defined (see for instance Run 18B in u-, v-, and w component) one may argue, that the energy source lies in a relatively well defined wave motion. This in view of the fact, that the research aircraft was deliberately dispatched into thermally stable regions with vertical (directional) wind shear. One would exclude, therefore, (random) convective motions with a positive contribution of energy from buoyant forces as likely energy sources. It seems rather, that stability and wind shears together would cause wave motion similar to gravity waves along a stable interface. As has already been shown by Helmholtz (1888, 1889, 1890) such waves become unstable and amplify exponentially if they are shorter than a critical wavelength which depends on wind shear and temperature gradient across the interface (see Haurwitz 1941, Reiter 1961, 1963b, c). Critical wavelengths of 2000 to 4000 ft are entirely within the range of possibilities offered by observed atmospheric structure. Gravity waves of this wave length range may be observed relatively frequently in cirrus, especially near the jet stream (Reiter and Hayman, 1962; Reiter and Nania, 1964).

Runs 18D, 33E, and 46E do not show a clearly defined "hump" in the w-spectra. These three runs were made under almost straight head wind conditions.[2] The obvious dependence of the shape of the spectrum curves on the angle between course and wind direction may be illustrated by the

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[2] Directions in Run 46E, made only a few hours after Run 45H, are considered with respect to directions of the strongest of the two winds measured across a shearing layer.

examples of Runs 18B and D which were obtained from the same turbulence patch. Run D was measured on a course heading straight into the wind, Run B on a course 65 degrees across the wind. No "hump" was observed in D, a pronounced "hump", however, was present in "B". This dependence of spectral density on flight direction supports the conclusion that a wave phenomenon of specific orientation with respect to the mean-wind direction was responsible for the observed energy input at wavelengths corresponding to the "hump".

Conclusions:

Project TOPCAT measurements of CAT power spectra over Australia suggest the following model of CAT formation which, in essence, confirms earlier hypotheses (Reiter 1960, 1963b, 1964e).

In stable layers of the upper troposphere and stratosphere, which contain sufficient vertical wind shear (measured in terms of vector wind shear), long wave perturbations tend to be anisotropic, showing a significant suppression of vertical perturbation components of motion because of the stabilizing action of negative buoyant forces.

CAT may be expected in such a stable environment, if critical wave lengths (below which the flow becomes unstable) are still above the ones equivalent to the CAT response frequencies of the aircraft. In the region of this critical wave length a significant amount of turbulent energy is made available to the flow through the vertical shearing stresses which counteract the effects of thermal stability. This may be expressed by the stability criterion derived from gravitational shearing waves

$$R = \Delta\rho - \frac{k\bar{\rho}}{2g}(\Delta\bar{u})^2 \;\begin{array}{c} < \\ > \end{array}\; 0 \quad \begin{array}{l}\text{unstable} \\ \text{stable}\end{array}$$

which bears similarity to Richardson's criterion (Reiter 1961, 1963c). $\Delta\rho$ is the dentisty difference and $\Delta\bar{u}$ the wind shear across the interface, k the wave number. As the latter increases, instability may be reached. Effects of shear on the formation of CAT through Richardson's criterion have recently been evaluated by Panofsky and McLean (1964).

At wave lengths shorter than the critical one the flow breaks down into isostropic turbulence. The CAT "bumps" themselves seem to be contained within this inertial subrange, although their cause should be sought in the energy initially released in a gravity wave-like phenomenon. The relatively high frequency of CAT observations over mountains and hills (Clodman et al. 1960) bears this out, too.

The apparent longevity of some CAT patches over Australia (see Table II, case of 12. 9. 63) suggests a rather stable mesostructure of the atmosphere to be present at times, which continuously supplies energy to the shorter waves in the inertial subrange. It may be of interest to note, that Run 33E (12. 9. 63) shows a "hump" only in the v-spectrum. The w-spectrum suggests an energy supply acting rather continuously at waves > 400 ft.

Although valuable evidence corroborating the above conclusions has been collected during Project TOPCAT the lack of exact data on vertical temperature gradients and wind shears in the CAT regions still is deplorable. The authors share the opinion expressed by Shur (1962) that careful measurements of supporting meteorological parameters are urgently needed.

Furthermore, it would be of great value, if the "smooth" regions surrounding a CAT patch were surveyed carefully, and power spectra were obtained from such less "exciting" measurement runs as well. It would be most interesting to investigate, for instance, whether the long-wave parts of the spectra remain the same inside and outside a CAT region, so that the energy increment supplied by the gravity-type wave formation would have to be held entirely responsible for CAT observed in a thermally stable environment. If this were the case, interface regions that might possibly harbour such wave formation, could be explored by various sensing techniques, such as backscatter of electro-magnetic waves.

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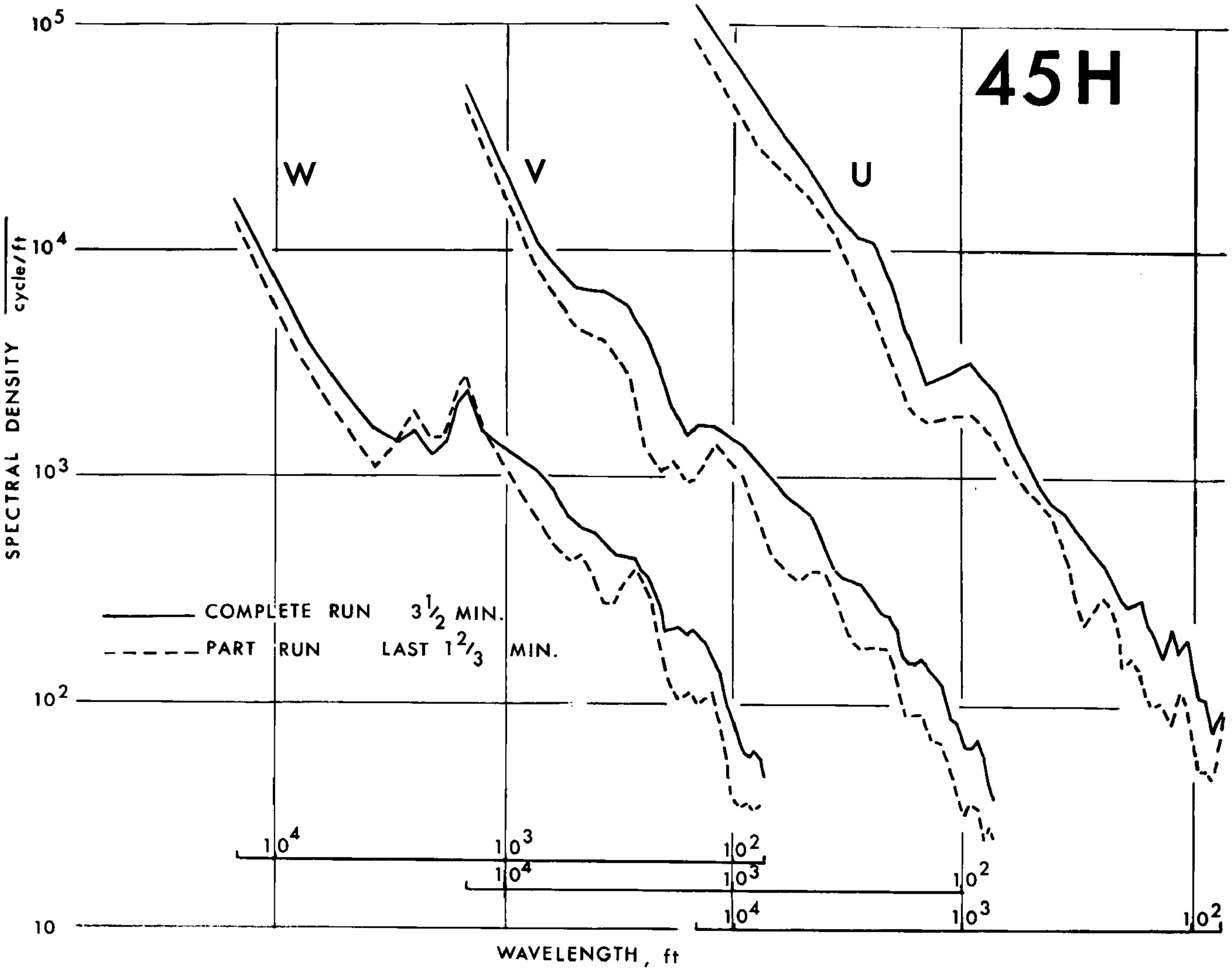

Fig. 1: Spectral densities of u-, v-, and w-components of gustiness, computed separately for last 1 2/3 minutes and for total flight time (3 1/2 minutes) of Run 45 H. (After Burns and Rider, 1965).

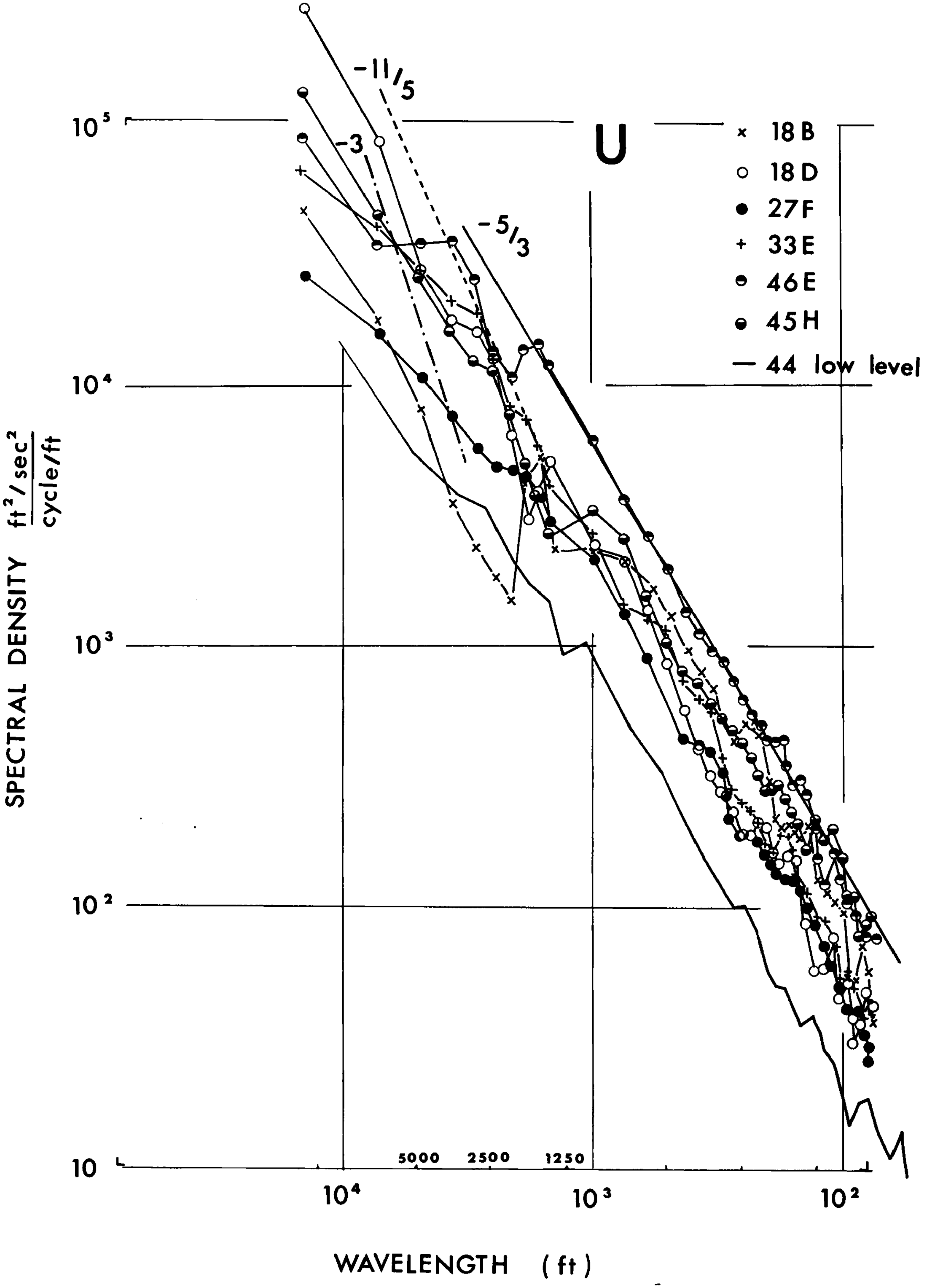

Fig. 2: Spectra of gustiness in u-component (along flight direction). Low level measurements and auxiliary "slope" lines are shown in red.

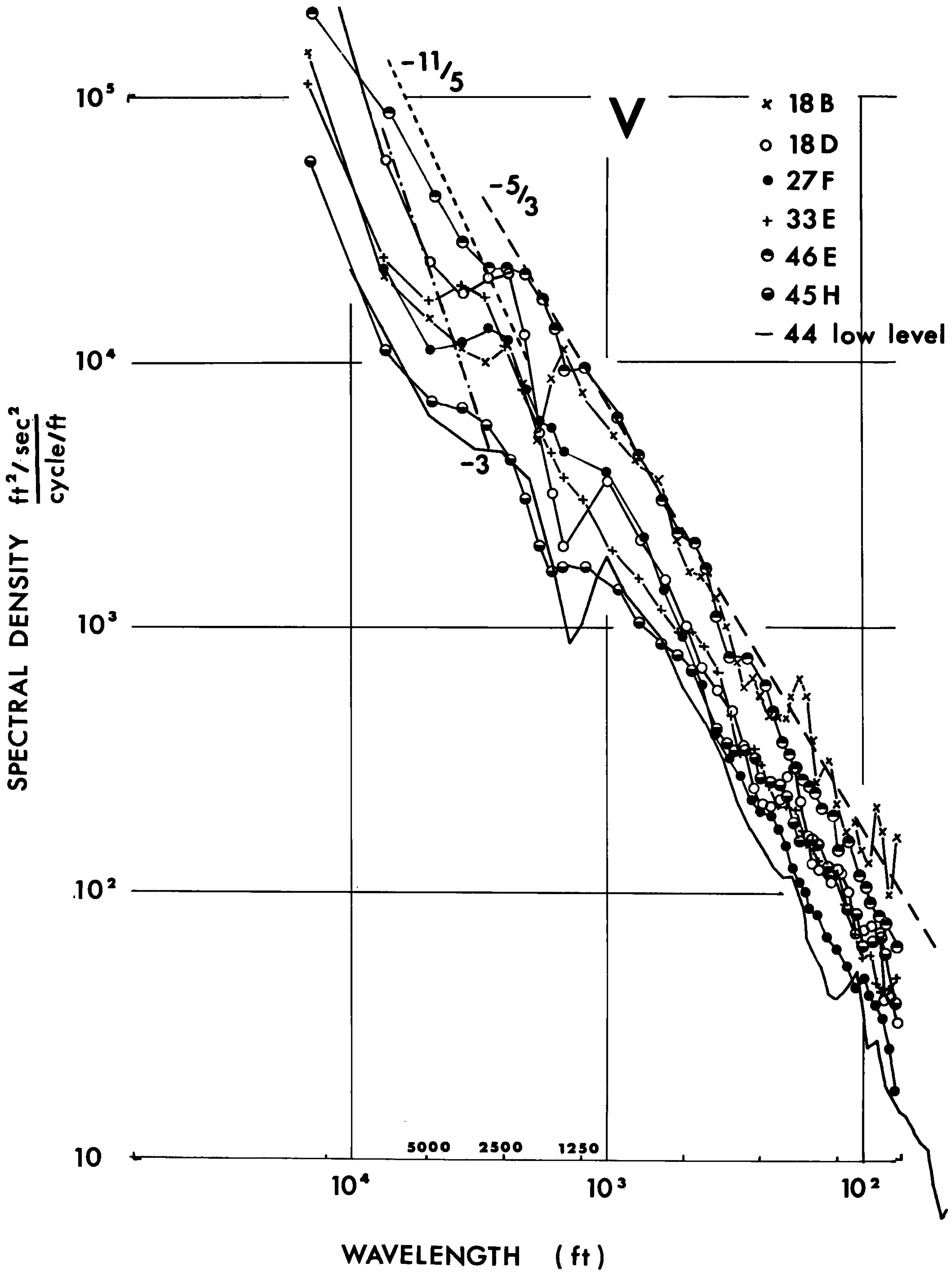

Fig. 3: Same as Fig. 2, except v-component (across flight direction)

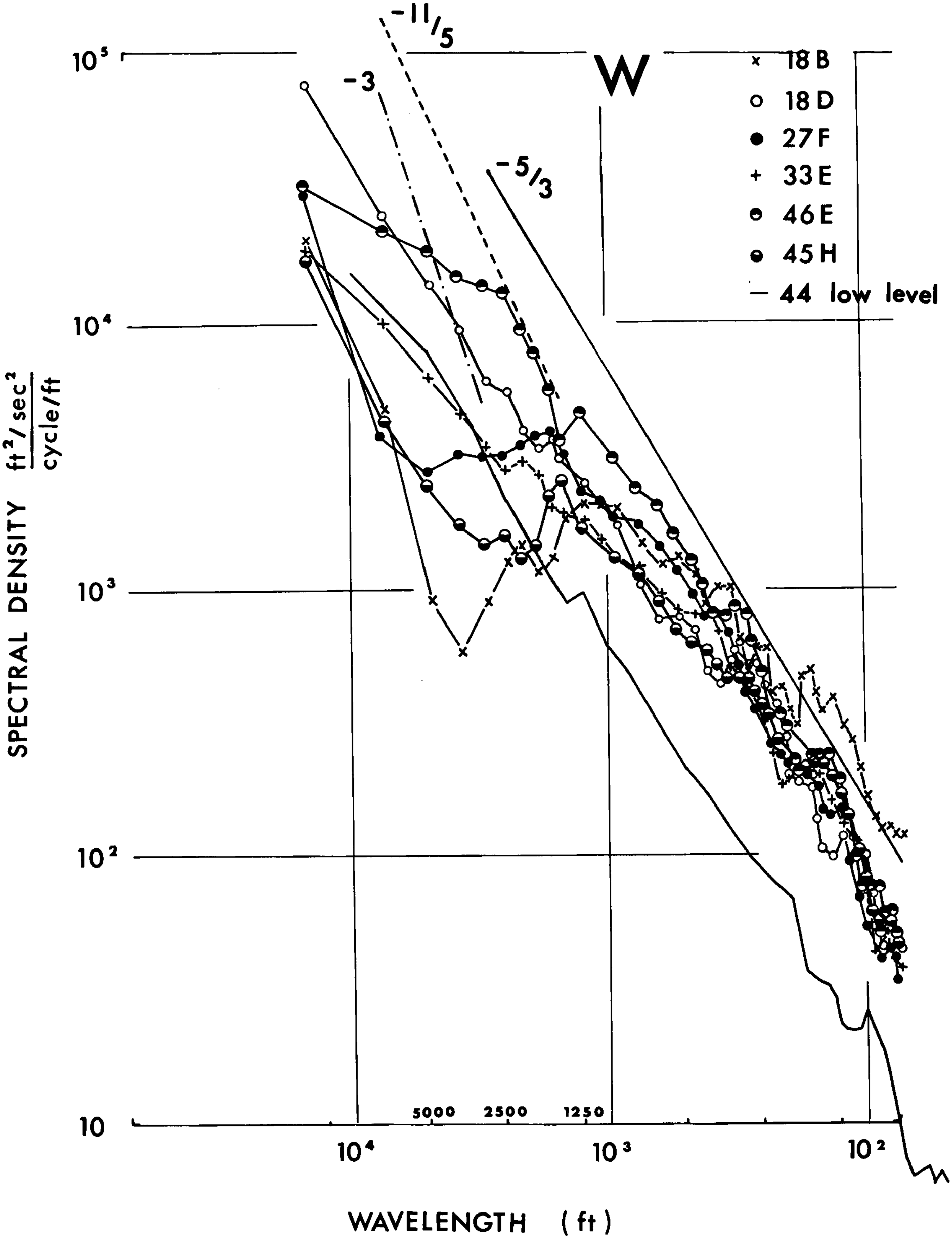

Fig. 4: Same as Fig. 2, except w - component.