Title: Comparing the effects of fuel treatment layouts in fragmenting large contiguous fuel patches under different fire duration assumptions

Abstract: Fuel treatment is an important component of wildland fire management. Fuel treatments can fragment large and contiguous fuel patches with high fire intensity potentials. This research applied a mathematical programming model to compare the effects of different fuel treatment layouts in fragmenting fuel patches, and controlling the future fire sizes under different fire durations assumptions. Analyses suggested that fuel treatment aimed at controlling fires of longer duration could effectively lower the risk of fires with shorter duration. However, fuel treatment layouts aimed at shorter fire durations might not perform well when the future fire duration is much longer. Fuel treatment layout designed under the assumption of infinite fire duration can effectively fragment high fire hazard fuel patches and provide reasonable support for future fire control.

Key words: wildland fire, simulation, optimization, fire duration

Introduction

Excessive fuels left from the long-term fire exclusion caused many forests prone to the risk of high intensity catastrophic wildfires across the western US. Fuel treatment can mitigate the risks of wildfires by influencing the fire burn probability and fire behavior (Parisien et al 2010) across a landscape. Reducing hazard fuels can slow fire spread, decreases fire intensity (Stratton 2004, Fernandes and Botelho 2003) and facilitates future fires control (Agee et al 2000, Hirsch et al 2004, Loehle 2004). The objectives of fuel treatment on a landscape include fragmenting fuel patches, changing wildfire size and behavior, and lowering the chance of fire spreading into wildland urban interface (WUI) (Finney 2001, Mell et al 2010).

Modeling for fuel treatment allocation can help improve the efficiency of hazardous fuel reduction programs (Salazar and Gonzalez-Caban 1987, Kaloudis et al 2005). Different fuel treatment locations collaborate across space to influence fire spread and intensity (Rytwinski and Crowe 2010). Some researchers suggested allocating treatments into parallel strips perpendicularly to major fire spread directions to better intercept fire spreads (Fujioka 1985; Catchpole et al 1989). Other researcher considered fuel treatment as a patch management problem and suggested fuel treatment layout should be designed to fragment high fire risk patches that have contiguous and heavy fuels (Agee et al 2000).

Decision support models were developed to schedule fuel treatments. Hof et al (2000) used a linear programming model to schedule fuel treatments to slow the movement of a specific fire to protect several preselected target locations. Bevers et al (2004) used a shortest path network model to study the effectiveness of random fuel treatment locations and suggested that a large portion of a landscape needs to be treated to form well-connected fuel breaks. Wei et al (2008) developed a mixed integer-programming (MIP) model to schedule fuel treatments to break fire probability accumulation pathways to lower the landscape fire risks. Kim et al (2009) compare fire spreads under dispersed, clustered, random, and regularly spaced fuel treatment layouts and suggested that fuel treatment may marginally lower wildfire severity during a severe fire season. Following the logic that fuel treatments need to be planned ahead to provide control for future fires (Pyne 1984) under various fire conditions (He et al 2004), Wei (2012) developed a two-stage model that schedules treatments to create fire control opportunities for many possible future fires.

This research implements the model developed by Wei (2012) to study how fuel treatments can be scheduled to fragment high fire hazard fuel patches. High fire hazard patches in the paper are defined as continuous areas of land that support high intensity fires. Patch management creates vegetation mosaics (Pinedo-Vasquez and Padoch 2001) that can fragment these patches. The contribution of this paper include: it designed a model revision to focus on fuel patch management; and it compared the effects of different fuel treatment layouts by simulating future fires with random durations.

Review and revise of a MIP model formulation

Previous research suggested fire spread in a landscape can be modeled by continuously tracing fire spread between adjacent cells through the minimum travel time (MTT) algorithm (Cheng and House 1996; Finney 2002; Sturtevant et al. 2009). Using the MTT algorithm, the fuel treatment optimization model developed by Wei (2012) selects treatment locations by modeling many future fires with predefined fire durations. This model is applied in a landscape delineated into many square raster cells. Fuel treatments are assumed to influence the fire intensity and rate of fire spread in a treated cell, and consequently alter the spread of many future fires. This formulation is reviewed here.

Set and Subscripts

C and r: the set and index of raster cells in a landscape.

C' and r': the subset and index of raster cells that have high fire intensity.

C'' and r'': the subset of raster cells that have low fire intensity. Fuel treatment is not required in these cells.

S and s: the set and index of raster cells from which ignitions could start.

Q_r *and q:* the set and index of cells directly adjacent to cell r (sharing an edge or a corner).

Parameters

B: the number of cells fuel treatment can be scheduled in.

D: the expected fire duration.

P_s: the probability of a fire igniting from cell s in the next discrete planning period.

L_r: value to be protected from fire in each cell r (loss if burned).

K: a positive constant denoting the delayed fire spreading time by fire control in a treated cell, or in any other cells with low fire intensity.

$\tau_{q,r}$: fire travel time from the center of cell q to the center of cell r without treatment.

Variables

$x_{r'}$: binary variable tracking treatment decisions in cell r'. We assume only cells currently having high fire intensity can be treated. $x_{r'} =1$ denotes that fuel treatment is scheduled in cell r'; $x_{r'} = 0$ denotes that no fuel treatment is scheduled in cell r'.

$x_{r''}$: denoting cell r'' currently having low fire intensity.

$t_{s,r}$: contiguous variable tracking the fire arrival time to cell r after ignited from cell s.

$y_{s,r}$: binary variable tracking whether fire will burn cell r within a duration D after ignited from cell s; $y_{s,r}=1$ denotes this fire will burn cell r; otherwise $y_{s,r}=0$.

Mathematical formulation

Minimize:

$$Z = \sum_{s\in S} \sum_{r\in C} P_s \times L_r \times y_{s,r} \qquad (1.1)$$

Subject to:

$$t_{s,s} = 0 \qquad \forall\, s \in S \qquad (1.2)$$

$$t_{s,r} \le t_{s,q} + \tau_{q,r} + K \times x_r \qquad \forall\, s \in S, r \in C, q \in Q_r \qquad (1.3)$$

$$y_{s,r} \ge \frac{D - t_{s,r}}{D} \qquad \forall\, s \in S, r \in C \qquad (1.4)$$

$$\sum_{r'\in C'} x_{r'} \le B \qquad (1.5)$$

$$x_{r''}=1 \qquad \forall\, r'' \in C'' \qquad (1.6)$$

Objective function *(1.1)* minimizes the total fire loss from all modeled fires within a predefined durations D. Loss caused by each fire is the total value loss within the fire footprint at the end of duration D. This loss is weighted by the probability of that particular fire ignition within the next discrete planning period (i.e. one year). Equation *(1.2)* sets the fire arrival time to cell s as zero when we assume fire is ignited from it. Fire will be ignited from every possible ignition cell on the landscape to help locate fuel treatments. Equation *(1.3)* applies the MTT algorithm to track the earliest time $t_{s,r}$ that fire could arrive at the center of cell r from any of its eight adjacent cells q after originated from cell s. The major fire spread direction in each cell represents the fastest fire spread direction (front fire) in that cell. Fires also spread along other directions at slower speeds as flank fires or back fires. We assume fire spreads in each cell following an elliptical shape (Green et al 1983) and the value of $\tau_{q,r}$ will be calculated using the major fire spread direction, distances between adjacent cells and the dimension of the ellipse reported by software such as FlamMap (Finney 2006). We assume cells with low fire intensity could delay fire spread due to the improved fire control efficiency. The amount of time delayed is defined by a parameter K. By setting the value of K larger than the modeled fire duration, we assume no fire would spread into the center of cell r if the fire intensity in it were low. Equation *(1.4)* defines binary variable $y_{s,r}$ working as a switch to track whether fire started from cell s would burn cell r within duration D. If fire reaches the center of cell r within duration D, then $D>t_{s,r}$, therefore $y_{s,r}$ will be set to one by Equation *(1.4)*; otherwise $y_{s,r}$ could be either zero or one. When given the choice (zero or one for $y_{s,r}$), the model will set it to zero to minimize the fire loss within duration D in objective function *(1.1)*. Equation *(1.5)* is a budget constraint reflecting the number of cells with higher fire intensity to be treated in the landscape. Equation *(1.6)* lets the model recognize that cell with low fire intensity should be considered as same as treated cell and can be used to delay fire spread without further treatment.

A revised formulation emphasizing patch management

An important objective of fuel treatment is to facilitate the future fire control. However, future fire conditions are often difficult to predict due to the impacts of stochastic factors such as fire duration and fire spread speeds along different directions. Fuel treatment layout optimized for a specific future fire condition might not provide the best control when the condition does not follow what is predicted. Putting more emphasis on patch management might help ease some of the challenges in accurately predicting the future fire conditions. The above mathematical programming model can be easily revised to meet the requirement of patch management. Only changes would be using new Equation *(1.3.2)* and *(1.4.2)* to substitute the original Equations *(1.3)* and *(1.4)*.

$$t_{s,r} \leq t_{s,q} + x_r \qquad \forall\, s \in S, r \in C, q \in Q_r \qquad (1.3.2)$$

$$y_{s,r} + t_{s,r} \geq 1 \qquad \forall\, s \in S, r \in C \qquad (1.4.2)$$

Using this new set of equations also eliminates the requirement of calculating parameter $\tau_{q,r}$ in Equation *(1.3)*, which saves us from predicting the major fire spread direction and the rate of fire spread in each cell. In reality these parameters are stochastic and difficult to be accurately predicted. This revised model focuses on the size, potential fire loss, and the fire ignition probability of the high fire hazard fuel patches in a landscape. Fragmenting high fire hazard fuel patches represents a decision problem that does not rely on accurate estimation of future fire spread direction and speed. After a fire is ignited from a cell s, $t_{s,r}$ will be set to zero following the logic built into equation *(1.3.2)* if cell r is in the same high fire hazard fuel patch as cell s; otherwise $t_{s,r}$ will be set to one indicating that fire will not spread from s to r. In equation *(1.4.2)*

$y_{s,r}$ will be allowed to be zero if $t_{s,r}$ is set to one, indicating that cell r will not be burned after a fire ignited from cell *s*.

The nature of this revised formulation can be described through an example. We can first set the probability of fire ignited in every cell to be 0.01, and set the value to be protected from fire in each cell to one for demonstration purpose.

N: the total number of disjointed fuel patches in the landscape after treatment. We will first assume N is a predetermined and arbitrary integer number for demo.

i: the index of each fuel patch after treatment.

M_i: the number of cells within each fuel patch *i*.

With constraint *(1.3.2)*, fire can start from each of the M_i cells in patch *i*, spread into all the other cells in the same patch, and cause a fire loss of M_i from each ignition. The model calculates the total expected fire size within each individual fuel patch *i* as $0.01 \times M_i^2$. The total expected future fire size from the *N* smaller patches becomes $0.01 \times \sum_{i=1}^{N} M_i^2$. This essentially requests that the MIP model to schedule fuel treatments to break a landscape into *N* disjoined smaller patches to minimize $\sum_{i=1}^{N} M_i^2$. Appendix *(1)* shows an example that, with a fixed *N*, the best way to minimize the value of $\sum_{i=1}^{N} M_i^2$ is to evenly distribute the number of cells into all smaller patches. In real world fuel management, the probability of fire ignited from each cell *s* (P_s), and the value to be protected from fire in each cell *r* (L_r) may vary across a landscape. The mathematical programming model will weigh the impacts of landscape heterogeneities when fragmenting large fuel patches. To further relaxing the assumptions, the value of N will also vary depending on the locations of fuel treatments. The mathematical programming model will also try to break the landscape into a larger number of high fire hazard fuel patches (larger N).

As we discussed before, solutions discovered by assuming infinite fire duration would not be sensitive to the changes of certain fire behaviors such as the rate of fire spread and the major fire spread direction in each cell. This is because when fire duration is long enough fire ignited in one cell will eventually spread into all other cells within the same patch regardless of the rate of fire spread and the major fire spread directions until this fire is stopped by fuel breaks. This forces the model to concentrate on landscape strategy to break fuel patches and ignore certain fire-spread details in each patch.

We tested four fuel treatment levels (seven-cell, eleven-cell, thirteen-cell, and twenty-four -cell) in an artificial landscape of 7×7 cells using the patch management assumption. These tests assume future fires will be completely stopped by treated cells. Allocating treatments into seven, eleven, thirteen and twenty-four cells each breaks the landscape into two, three, four or nine disjointed smaller fuel patches regardless of the specific rate of fire spread and major fire spread direction in each cell (Fig. 1).

Figure 1 approximately here

A test case

A small portion of Sequoia and Kings Canyon National Parks (SEKI), with an extent of 3.6 by 3.6 km is used as a test example here. This landscape is rasterized into four hundred 180m wide raster cells. We first tested the original mathematic formulating by assuming a prevailing southwest wind at eight km per hour with moderate understory fuel moisture condition. FlamMap is used to quantify fire behavior in each cell including the major fire spread direction, the fire flame length, the dimension of the burn ellipse and the rate of fire spread. For comparison, we also run the revised version of the model with the focus on patch management. This revised version of model does not require the fire behavior data such as spread speed and spread directions. In this example, a 2.44 m (eight feet) flame length threshold is used as the example to identify cells that can benefit from fuel treatment. All cells are classified into two categories based on this threshold.

Figure 2 approximately here

The value to be protected from fire in each cell is assumed to depend on the presence of WUI (with a value of one per cell) and other forests (0.4) within that cell. These values vary between locations across the study site (Fig. 2a). The annual ignition probability assigned to each cell (Fig. 2b) is given by Equation *(1.7)*

$$A_r = P_r \times \frac{\alpha}{Y \times \pi \times R^2} \qquad \text{------ (1.7)}$$

The annual ignition probability is denoted by A_r. P_r denotes the recorded number of ignitions obtained from historical records during the past $Y=83$ years, where r is the index of raster cells. α is set to be 180×180 = 32,400 m^2, which is the size of each raster cell. πR^2 is used to estimate area of a circle with the radius of R. The circle area is to be used in the Kernel density estimation, which is a tool to estimate the average ignition density of each raster cell in ESRI@ ArcMap. We used $R=290m$ as the rule of thumb. Larger R value will further smooth the spatial variation of ignition probabilities.

Figure 3 approximately here

Treatment under three fire duration assumptions

To compare the fuel treatment effectiveness under different fire duration assumptions, the model is applied to treat eight cells under three different fire duration assumptions: (1) Infinite fire duration, (2) a shorter (320minutes is used here) fire duration, and (3) a longer duration（24 hours. Scheduling fuel treatment under the assumption (1) would only require us to identify cells with potential high fire intensity; while running the model under assumption (2) or (3) requires more detailed fire behavior data such as the rate of fire spreads along different directions in each cell.

Without fuel treatment, there are currently eight high fire hazard fuel patches composed by cells with potential high fire intensity. The size distribution of these patches is described in Table 1. After treatment in eight cells, the 20×20 landscape is fragmented into smaller and disconnected high fire hazard fuel patches. For planning scenario under the assumption that each fire lasts for infinite fire duration, a fire ignited from a cell would spread to its neighborhood cells and eventually to all other connected cells until they extinguish. To model under this assumption, Equation *(1.3.2)* and Equation *(1.4.2)* will be used to substitute Equation *(1.3)* and *(1.4)*. Model suggests an optimal fuel treatment layout as shown by Figure 3a and 3d. Under the planning scenarios of shorter fire duration (D is set to 320 minutes in Equation *(1.4)*), the model designs the optimal eight-cell fuel treatment layout as shown in Figure 3b and 3e. For planning scenario with longer fire duration (24-hr duration), the optimal fuel treatments allocations are given in Figure 3c and 3f.

Table 1 approximately here

Directly studying the distributions of high hazard fuel patches reveals the difference of optimal spatial fuel treatment patterns due to the change of future fire duration assumptions. In Table 1, the 20×20 landscape without treatment is comprised of eight disjoined high fire hazard fuel patches, with the largest patch of 599-hectare in size. In the case of using the 320- minute fire duration assumption, after eight cells are treated, the landscape still has eight high fire hazard fuel patches. However, the largest one is twenty-six hectare smaller than the largest patch before treatment. When scheduling treatment under the assumption of the 24-hour fire duration, the number of high fire hazard fuel patches increases to thirteen, and the size of the biggest patch decreases to about half of the largest patch before treatment. In general, this model prefers treatment layouts that can create more patches under longer fire duration assumption in contrast with shorter fire duration. When the fire duration is assumed to be infinite, this model would still schedule fuel treatments to fragment the landscape into thirteen patches with the largest patch of 288 ha. The patch distribution between 24-hour duration and infinite fire duration are very similar.

Figure 4 and Table 2 is approximately here.

A method to evaluate the treatment effectiveness

To further compare the three fuel treatment layouts, we evaluated the effectiveness of them against six random fire duration scenarios. A simulation algorithm that relies on the repeated random assignment of fire durations to each ignition is used in the test. A flow chart (Fig. 4) helps illustrate this testing process. The three fuel treatment layouts (corresponding to the 320-minute, 24-hour, and the infinite fire duration assumptions) are tested against each of the six random fire duration scenarios. The random fire durations range from (0, 360) minutes up to (0, 14,400) minutes (Table 2) depending on the specific scenario tested. For each fire duration scenario, thirty replications are generated to provide statistical estimates. In each replication, fires are ignited from all possible locations (cells) in a landscape with their duration randomly fluctuating within the range defined by the specific scenario. The expected loss from each fire is calculated by multiplying the ignition probability of that fire and the fire loss from that ignition and following spread. The total fire loss from all these fires will be summarized for each replication. After that, the mean expected fire loss of thirty replications under one specific random fire duration scenario is recorded. The effects of fuel treatment layouts in stopping future fires' spread and decreasing fire loss under each random fire duration scenario are then summarized and compared in Table 2 by using Tukey's test. This test compares the means fire loss under every treatment to the means of every other treatment. The formulations are listed below.

$$SE = \sqrt{\frac{MSE}{n}} \quad (1.8)$$

$$q_{observed} = \frac{G_i - G_j}{SE} \quad (1.9)$$

Equation *(1.8)* calculates *SE*, the stand error of all the samples used to study each tested stochastic fire durations scenario. *MSE* is the mean square error of all samples, and *n* is the number of observations collected from one treatment layout when it is tested against one stochastic scenario (n = 30 in this example). Equation *(1.9)* calculates a $q_{observed}$ value to identify where the difference between the two means is significant. G_i and G_j are the group means being compared, the confidence coefficient α is set to be 0.05 in our test case. In this way, we compare and rank the effectiveness of three fuel treatment allocations.

According to the test results summarized in Table 2, if all simulated fires last for less than 360 minutes, none of the fuel treatment layouts would perform significantly better than any of the other two fuel treatments. However, fuel breaks scheduled for fires lasting for 24-hr or for the infinite duration are significantly more effective in controlling long duration fires. Based on the testing results, fuel treatments aimed at controlling fires of longer duration could still perform well to lower the risk of fires with shorter duration. However, treatments' spatial arrangements aimed at controlling the shorter fire durations often not perform well when the duration of future fire is much longer.

Discussion and conclusion

This research provides preliminary tests of implementing a mathematical programming model to break contiguous fuel patches. This model ignites and grows a fire from every possible fire ignition cell in a landscape. It weights the loss caused by each fire by the probability of corresponding fire ignition. When the modeled fire duration is long enough to allow any fire spreading into all cells within the same contiguous fuel patch, this model can schedule treatments to efficiently fragment contiguous fuel patches to decrease the future fire loss. Patch oriented fuel management accounts for the spatial distributions of fire ignitions and the values to be protected from fire. It helps design treatment strategies to prevent excessive fire loss when the future fire duration is difficult to predict. Simply managing fuel patches creates consistent solution over a range of possible fire behavior assumptions. Patch oriented fuel treatment designs could miss certain fine scale fire behavior details, but would be less demanding on the future fire behavior predictions, which may be difficult in some real world fire management situations.

Patch related modeling not only can be used in breaking large patches of hazard fuels, but also might be implemented in preventing the spread of other detrimental disturbance agents such as insects and diseases, or invasive species etc. Patch management has represented a challenging decision problem due to the possible spatial variations of size, shape, connectivity and location of patches. This model could be used to fragment patches and potentially be extended to support other related research.

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Appendix 1: An example of how to best allocate β cells into N disjointed patches to minimize cell interactions in case of fire spreading between all cells in each patch.

Minimize $\quad M_1^2 + M_2^2 + M_3^2 + \cdots + M_N^2 \qquad$ *(A.0)*

St: $\quad M_1 + M_2 + M_3 + \cdots + M_N = \beta \qquad$ *(A.1)*

$M_1, M_2, M_3 \ldots\ldots M_3 \geq 0$

This problem can be solved through Lagrangian method. We can use v to denote the shadow price of equation *(A.1)*

$$\mathrm{L}(M_i, \mathrm{v}) = M_1^2 + M_2^2 + M_3^2 + \cdots + M_N^2 + v \times (\beta - M_1 - M_2 - M_3 - \cdots - M_N)$$

At the stationary point, both $\frac{\partial L(M_i,\mathrm{v})}{\partial M_i} = 0$ and $\frac{\partial L(M_i,\mathrm{v})}{\partial v} = 0$, therefor:

$$M_1 = M_2 = M_3 \ldots = M_N = v/2$$

Table 1. The distributions of high fire hazard fuel patches with different fuel treatment layouts that are based on various fire duration assumptions.

Seq. No. of hazard fuel patches sorted by patch area	Patch size without treatment (ha)	Patch size after treated 8-cell under various fire duration assumptions		
		320-min duration	24-hr duration	INF duration
1	599	573	288	288
2	10	10	156	146
3	6	6	97	97
4	3	3	16	26
5	3	3	10	10
6	3	3	10	10
7	3	3	6	6
8	3	3	6	6
9			3	3
10			3	3
11			3	3
12			3	3
13			3	3

Table 2: Comparing the efficiency of different fuel treatment layout by simulating fires from all possible fire ignition locations in a landscape. The simulation duration of each fire is assigned through random drawn.

Stochastic Scenario Seq.	Simulation duration used for post treatment evaluation (minutes)		Expected fire loss after fuel treatments scheduled under different fire duration assumptions			Comparing fuel treatment effectiveness using the Tukey's method (95% confidence)
	From	To	320-min	24-hr	Inf	
1	0	360	18.0	18.2	18.1	No solution is better
2	0	720	33.8	25.0	25.0	24hr>320min,INF>320min
3	0	1800	88.6	31.0	30.2	24hr>320min,INF>320min
4	0	3600	144.0	33.4	32.7	24hr>320min,INF>320min
5	0	7200	178.2	34.8	33.8	24hr>320min,INF>320min
6	0	14400	195.4	35.4	34.4	24hr>320min,INF>320min

A > B means solution based on duration A is significantly better than solution based on duration B

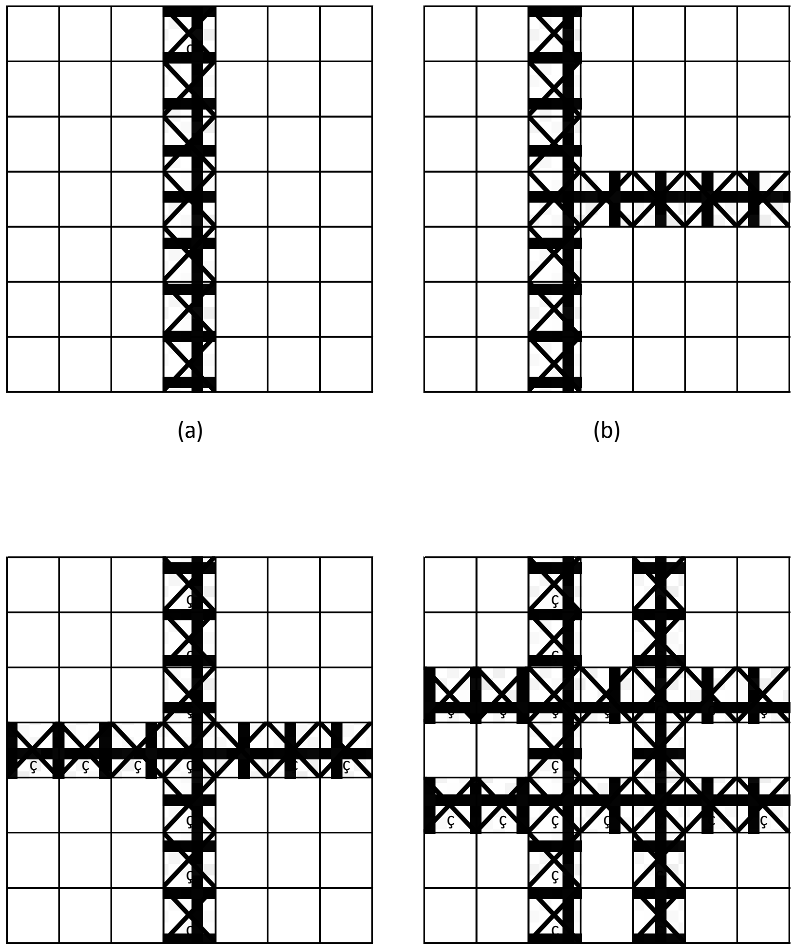
Figure 1: Four fuel treatment levels (seven-cell, eleven-cell, thirteen-cell, and twenty-four -cell) in an artificial landscape of 7×7 cells using the patch management assumption. These tests assume future fires will be completely stopped by treated cells.

Figure 2: (a) The value to be protected from fire in each cell is assumed to depend on the presence of WUI (with a value of one per cell) and other forests (0.4) within that cell. (b) The annual probability of fire ignited from each cell is calculated using historical records.

Figure 3: Different optimal spatial fuel treatment patterns under different future fire duration assumptions. (a) and (d) show the eight cells selected for treatment, the high fire hazard fuel patches, and the expected fire loss for fire ignited from each cell using the assumption of infinite future fire duration. It represents the result of patch oriented management. (b) and (e) represent the solution from the 360-minitue fire duration assumption. (c) and (f) represent the solution from the 24-hr fire duration assumption.

Figure 4: A flow chart helps illustrate an algorithm to evaluate the effectiveness of different fuel treatment layouts against random fire duration scenarios. This algorithm relies on the repeated random assignment of fire durations to each ignition.

Figure 1



(a) (b) (c) (d)

Figure 2

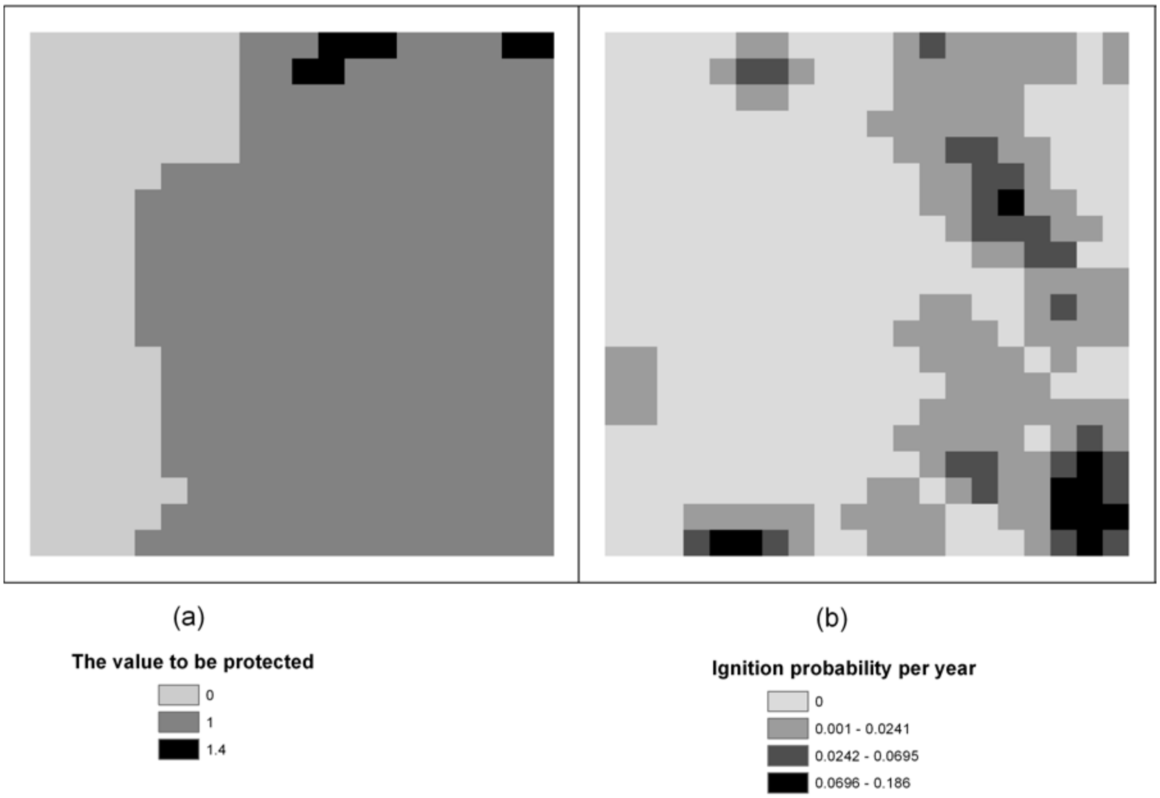

(a)
(b)
The value to be protected
0
1
1.4
Ignition probability per year
0
0.001 - 0.0241
0.0242 - 0.0695
0.0696 - 0.186

Figure 3

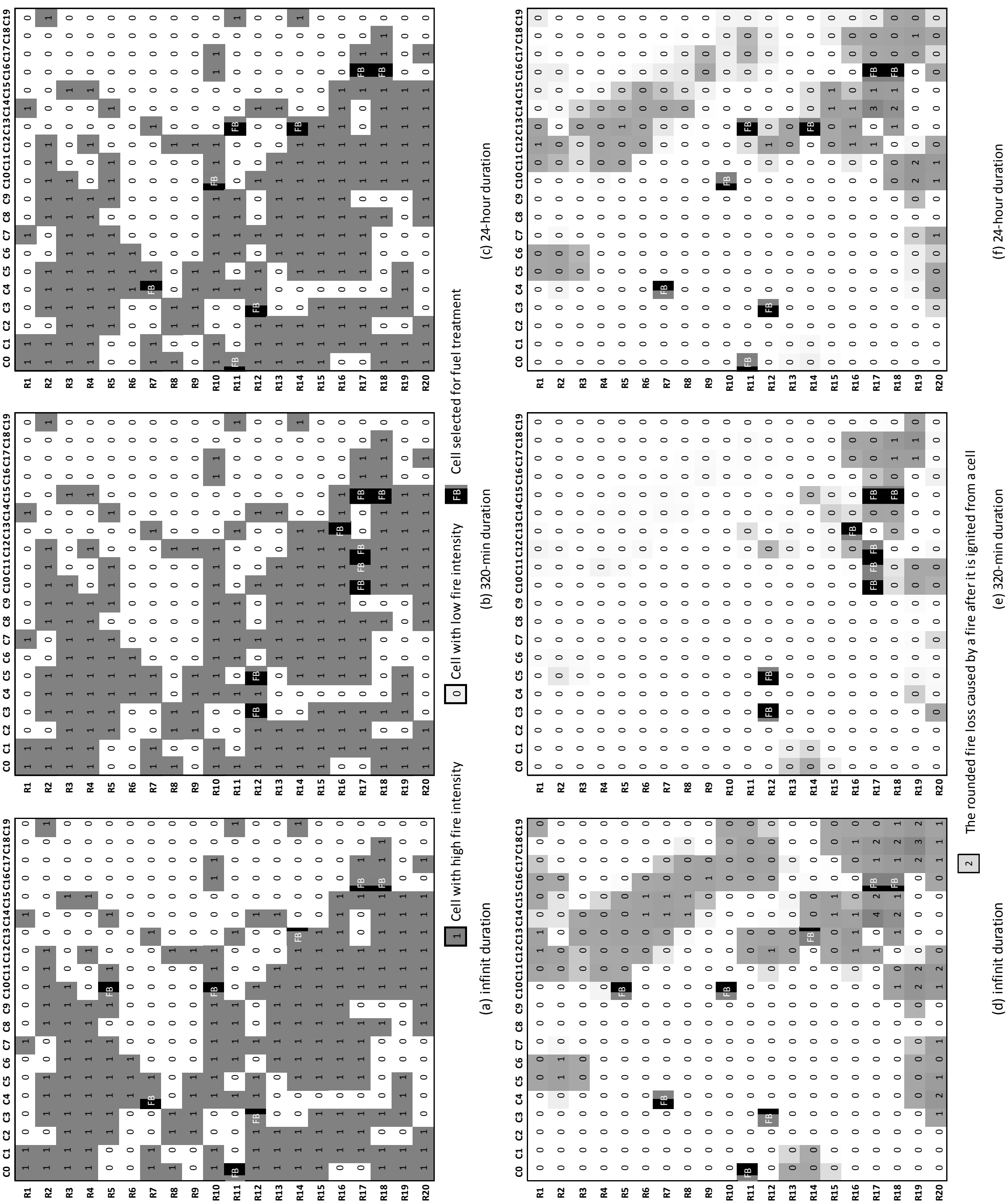
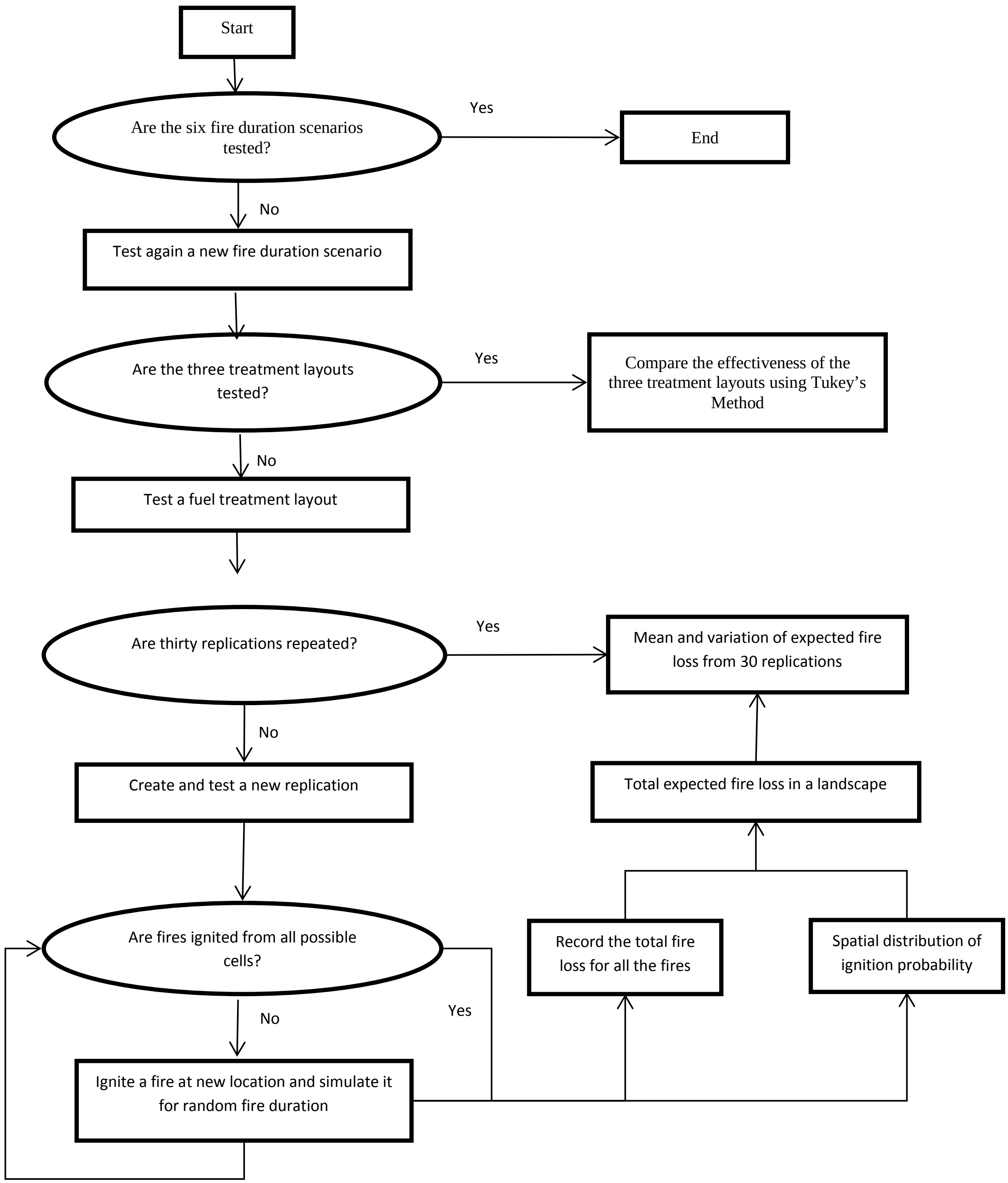

(a) infinit duration
(b) 320-min duration
(c) 24-hour duration
(d) infinit duration
(e) 320-min duration
(f) 24-hour duration
Cell with high fire intensity
Cell with low fire intensity
Cell selected for fuel treatment
The rounded fire loss caused by a fire after it is ignited from a cell

Figure 4


Start
Are the six fire duration scenarios tested?
Yes
End
No
Test again a new fire duration scenario
Are the three treatment layouts tested?
Yes
Compare the effectiveness of the three treatment layouts using Tukey's Method
No
Test a fuel treatment layout
Are thirty replications repeated?
Yes
Mean and variation of expected fire loss from 30 replications
No
Create and test a new replication
Total expected fire loss in a landscape
Are fires ignited from all possible cells?
Record the total fire loss for all the fires
Spatial distribution of ignition probability
No
Yes
Ignite a fire at new location and simulate it for random fire duration