

DISSERTATION

BRAIN DRAIN AND REVERSE BRAIN DRAIN: INDIVIDUAL DECISION
MAKING AND IMPLICATIONS FOR ECONOMIC GROWTH

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ABSTRACT

BRAIN DRAIN AND REVERSE BRAIN DRAIN: INDIVIDUAL DECISION MAKING AND IMPLICATIONS FOR ECONOMIC GROWTH

In two models, this dissertation explores two different but related topics in the international migration of skilled individuals, namely, the possibility of beneficial brain drain arises from the out-migration of skilled individuals and the potentially economic incentives for the emigrants to return to their homeland.

The first model is a R&D and human capital accumulation hybrid endogenous growth model with a modified human capital accumulation behavior. It shows that an individual learns from previous innovations and that human capital accumulation fuels improvement in the quality of goods to promote economic growth. Since the ability of an individual is the key to the formation of human capital, the economic growth rate is tied to the representative individual's ability. It also shows that, with the presence of uncertainty about the opportunity of migration, the sending country could benefit from brain drain even without the scale effect.

The second model is a two-period overlapping generation human capital growth model with a common self-selection fashion. Each individual optimally chooses his human capital level and the location he works. It shows that an increase

in the probability of migration induces human capital accumulation in the sending country resulting from more individuals becoming potential returnees, and each potential emigrant or returnee acquiring more education. It also shows that the domestic investment opportunity could further increase human capital acquisition for a potential returnee while the wage premium couldn't.

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1 INTRODUCTION

1.1 Background

Since the 1960s, Taiwan sent out hundred thousands of highly intelligent students, those who could be the future of Taiwan, to enroll in the top universities in advanced countries such as US, Germany, and Japan. In the 1970s and 1980s, 20% of undergraduates in the field of science and technology went abroad for higher education (Chen, 1995). Taiwanese expected that they could bring back valuable knowledge. In the 1970s, driven by small businesses, Taiwanese economy boomed. However, most of these students did not participate in this economic take-off. Only about 16.2% of them returned after graduation in 1977 (Su, 1995). Brain drain became even worse in subsequent years. In 1979, it was registered that only 8.2% of them returned. What happens was that they found jobs and lived in those countries. The opportunities for them to succeed in high-tech industries were still not high in Taiwan. Hence, the economic incentives for them to return were still too low. It seemed not so wise for the government to send them out at that time.

The need for upgrading industries in the late 1970s emerged. Taiwanese government recognized the situation and became more active in reversing the brain drain. In late 1980, Government built Hsinchu Science Park and promoted tax reductions to introduce high-tech industries, such as I.T. industry. Suddenly, the opportunities for highly skilled emigrants expanded tremendously. Taiwan was calling! The land of impossibilities became the land of possibilities. Like salmon swim back to where they were born, many foreign trained scientists and engineers flied back to their motherland. They devoted their human capital

and physical capital accumulated abroad in exploring opportunities. They became leaders in research and development, the partners, and even the owners of companies. Industries were upgraded and the economy further boomed. Consequently, more skilled emigrants returned. In the late 1980s, it was estimated that one-third of foreign educated students returned (Su, 1995). From the survey based on the Taiwan 1990 population census, around 50,000 returned during the period 1985-1990. It turned out that knowledge acquired in advanced countries awaited the right moment to be passed on. Brain drained and then brain gained; it was a full circle of brain circulation.

This was not a unique situation for Taiwan. Several developing countries are experiencing serious economic impacts of brain drain. India is another typical example. After decades of brain drain, India is ready to reverse the brain drain by establishing its own science parks targeting the drug industry. In the 1990s, the main drug companies were striving to pirate drug formulae. For example, Cipla and Ranbaxy Laboratories were selling one-dollar AIDS cocktails in India and Africa. However, Indians knew that it was a dead end by coping. In order to upgrade the drug industry, they must equip with the ability to do research and development. In 2003, India announced that they would protect the rights of foreign patent holders. Lacking biologists, drug companies in India collaborated with western drug companies in performing fairly simple lab work and accumulating experiences. In order to expand their biology capability, they started to attract Indian-born biologists back. In 2006, India began to issue overseas citizen of India cards, offering foreign citizens of India origin visa-free entry for life and work in the country. By July 2008, more than 280,000 cards had been issued. A brain circulation has been complete.

1.2 Outline of Research Questions

Two issues are pertinent here. The first issue is the possibility of a positive impact of brain drain on the sending country's economy. Common wisdom implies that brain drain damages the sending country's economy and benefits the receiving country's economy. Recent

literature, such as Beine et al. (2001), suggests that brain drain could also lead to a gain from personal schooling decisions. When the sending country opens for emigration, induced by higher returns to human capital in the receiving country, an individual who is planning to emigrate could acquire relatively more human capital. Some individuals might fail in emigrating. These individuals who fail to obtain visas could make contributions to the sending country by increasing the overall human capital level. Modern economic growth theory suggests another channel of economic growth through research and development (R&D). In my knowledge, none of the brain drain literature mentions, whether brain drain leads to a gain or a loss for the sending country in the context of R&D is still unknown. Hence, the first model in this dissertation is constructed to examine the possibility of beneficial brain drain in the course of integrated R&D and human capital growth. Another issue that arises is the factors that complete the brain circulation. In the brain drain literature, works, such as Borjas and Bratsberg (1996) and Mayr and Peri (2008), recognize wage premium as the economic incentive that attracts highly skilled emigrants back to the sending country. From the section 1.1, we learn that the improvement of economic environment could be another reason that reverses the brain drain. This area of study is still missing. Hence, the second model in this thesis is constructed to examine the possibility of the completion of brain circulation through the improvement of economic environment.

1.3 Outline of the Dissertation

This dissertation explores two different but related topics in the international migration of skilled individuals, namely, the possibility of beneficial brain drain which arises from the out-migration of skilled individuals and the potentially economic incentives for the emigrants to return to their homeland. It is organized in six chapters. The next chapter, chapter two, reviews the literature on brain drain, reverse brain drain, and economic growth. It begins from the historical evolution of the brain drain literature. Then, it turns to the economic growth literature. Several well-known works are presented. It finishes with the

reverse brain drain literature. Chapter three provides the first theoretical model and its applications. An integrated innovations and human capital accumulation model is constructed as a benchmark model. From this, a model with a modified human capital accumulation behavior is presented. Chapter four discusses the second model. Following other brain drain works, it is built on the self-selection foundation. Under this framework, individual decision toward economic incentives, including wage premium and the opportunity to succeed, is considered. Chapter five performs the numerical experiments. Due to the limitation of data, calibrations of these two models are adopted. In the final chapter, chapter six, a brief summary of the findings is given.

2 LITERATURE REVIEW

The purposes of the dissertation are twofold. The first purpose is to identify the possibility of a beneficial brain drain in a course of integrating R&D on one hand and human capital accumulation growth on the other. The second purpose is to identify the factors that contribute a brain drain - reverse brain drain circulation. In serving the first purpose, two classes of literature are under review. One class of literature is the traditional brain drain literature, a branch of the migration literature. It discusses the impacts of the skilled labors emigrating to the receiving country on the overall human capital stock of the sending country. The endogenous growth literature focuses on the relationship between the human capital accumulation, the technological progress, and the long-term economic growth. For the second purpose, the reverse brain drain literature is reviewed.

2.1 Brain Drain Literature

The earliest work of brain drain can be traced back to Grubel and Scott (1966). This paper along with other first wave of brain drain literature including Johnson (1967) and Berry and Soligo (1969) set up the models in the perfect competition manner. Without any type of imperfections, all markets clear at all time and hence there are no welfare impacts on the sending country.

It was Bhagwati and Hamada (1974) that started to develop more realistic second-generation models that contain market imperfections. They worked in a general equilibrium framework with two types of imperfections, namely, the labor market wage rigidity

and unemployment, and the distortion in the finance of education in the sending country. And their result is that the emigration of skilled labors may reduce the overall productivity and wages in the sending country, and consequently, the remaining residents in the sending country encounter a welfare loss.

Like the work of Bhagwati and Hamada (1974), the subsequent works on the brain drain continued to incorporate market imperfections into their models. Hamada and Bhagwati (1975) extended their previous work to include imperfect information about the quality of labor. McCulloch and Yellen (1975) distinguished between skilled and unskilled labors and assumed that only the skilled labors in the less developed country migrate to the more developed country in a static model in which there is no capital accumulation. Rodriguez (1975) further modified these models into a dynamic model with physical capital accumulation in order to discuss the transitional and steady-state behavior of such an economy. Blomqvist (1986) presented a more general model in which Grubel and Scott (1966) and Bhagwati and Hamada (1974) are special cases.

Twenty years later, the endogenous growth theory was first introduced. Miyagiwa (1991) investigated the skilled labor migration issue in the context of human capital accumulation growth theory. He allowed individuals to choose whether to obtain education, and whether to emigrate¹. However, an individual can only choose either to obtain education or not. He cannot choose the amount of education. In this way, the economy contains two classes of labor – educated (skilled) and uneducated (unskilled). Under the assumption of scale economy, the more educated are laborers in a country the higher productivity of each individual, and, more people in the less developed country are willing to obtain education and emigrate to the more advanced country for higher wage rates.

Haque and Kim (1995) approached this issue by presenting an overlapping generation growth model where heterogeneous individuals live for two periods. When an individual is young, he can choose the allocation of time between education and work. The amounts of

¹That is self-selection.

human capital acquired are different across individuals. As he gets old, he can only work. Galor and Tsiddon (1997) presented a similar model but with three periods. In the first period, an individual invests in education by borrowing money from the financial market. In the second period, he works with all the human capital acquired in the first period. This model also allowed for different human capital acquisitions. In the third period, he retires and supplies all the saving to the financial market. The mechanisms of these two models are similar. By choosing the allocation of time, an individual maximizes his lifetime utility. Because the models assumed that abilities are different across individuals, they allowed for different individuals to have different levels of human capital. When the economy opens for migration, an individual knows that he could earn more if he migrates to a more advanced country. If his benefit of migration is greater than his cost of migration, he will move to this country. In this way, those who migrate to the more advanced country are those individuals with high human capital stocks. Of course, the overall human capital stock for the sending country decreased after out-migration.

Assumptions about the intergenerational spillovers of human capital are different in these two works. In Haque and Kim (1995), each individual inherits the average level of human capital from the previous generation. Each individual is unique in that his ability to learn is different from others. Galor and Tsiddon (1997) assumed that an individual completely inherits his parent's human capital level.

In spite of different assumptions and structures, the main findings are the same among the second-generation brain drain models. They all found that skilled migrants could lead to an overall human capital stock reduction in the sending country. This is the so-called brain drain effect. So, they took a more pessimistic view toward the skilled international labor migration.

Recent works, the third generation of brain drain literature, revealed another force of brain gain effects working in the opposite direction. Uncertainty about the opportunity to migrate could lead to an overall higher education attainment and human capital stock for

the sending country. One can expect that the receiving country could accept some and reject others of the immigration applicants. Mountford (1997) extended both Miyagiwa (1991) and Galor and Tsiddon (1997) to allow the host country the control of the number of visas issued and hence creating uncertainty for the potential immigrants. Vidal (1998) further extended Mountford (1997) to endogenize the probability of immigration – it depends on the average level of the sending country’s human capital. Beine et al. (2001) allowed for individuals to determine the amount they want to invest in education to enhance the opportunity of their migration. Docquier et al. (2007) created an endogenous human capital model with physical capital accumulation. Stark et al. (1998) specified the conditions under which this situation could happen.

The key arguments behind these works are the same. Because an individual can choose the amount of education, he can choose the amount of human capital to acquire. While facing the opportunity to earn a higher reward from human capital accumulation in a more advanced country, he would obtain more human capital than he would without the opportunity. In the case of uncertainty about migration, some successfully immigrate to the receiving country while others stay in the sending country. Those who succeed tend to reduce the overall human capital level, a brain drain effect, and those who fail tend to increase the overall human capital level, a brain gain effect for the sending country. As described by the second generation of brain drain literature, with certainty about migration, skilled labor migration thus leads to a welfare loss for the sending country. There is only the brain drain effect. Here, in the third generation, with uncertainty, there is a brain gain effect as well. If this brain gain effect could dominate the brain drain effect, it is possible for skilled labor migration to lead to a welfare gain for the sending country.

2.2 Endogenous Growth Theory

Paul Romer published his path-breaking paper – Endogenous Technological Change in 1990. He was the first one to successfully explain the reason why technological progress

leads to economic growth. He also elaborated the properties of technology. Both of them have become the new standard framework of endogenous growth models.

Romer started by distinguishing two fundamental properties of technology. A new technology consists of two components. The first component is the idea of knowledge. Because of the zero marginal cost of knowledge spillovers, technology has the non-rival characteristic of a public good. The second component is the product itself. Since people can block others from imitating their ideas and producing the same products by intellectual property rights, technology is excludable, at least partially.

The micro-foundation of Romer's model incorporated these two properties. Because of the excludability, economic agents can block the entry to the product markets. Once an economic agent discovers a new idea and creates a new product, this agent can legally force others not to produce this new product without his authorization. This agent thus can produce and sell this product with monopoly power. It is this profit opportunity that drives economic agents to invest in R&D and create new products. It is this power that drives technological progress. As the technology progresses, the number of product markets in the economy becomes larger and larger. This is the so-called variety expansion type of innovation or horizontal innovation.

Because of the non-rivalry characteristic, all economic agents in the economy can efficiently access to the entire stock of knowledge of technology. As the stock grows or as the amount of human capital invested into R&D increases, the creation of new technology grows. The new creation of technology is a function of the total amount of human capital invested into invention and the stock of technology. For simplicity, Romer (1990) modeled the "technology production function" in a linear fashion. Later on, one would see the reason why this assumption could lead to a problem in fitting empirical observations.

Grossman and Helpman (1991) and Aghion and Howitt (1992) inherited the same spirit but took another approach. They recognized that sometimes, quality-updating innovations could increase total factor productivity and hence improve standard of living. Researchers

try to find ways to improve the quality of some existing products (vertical innovation or quality improvement) instead of trying to find new products (horizontal innovation). In this type of economy, only the firm that produces the highest quality product can enjoy monopoly rents because a new improved quality product replaces the existing one and takes over its market and no economic agents are willing to purchase lower quality products². Economic agents compete to improve the quality of products and try to be the leader in the industry. Like Romer (1990), the technology production functions in these models were constructed as linear functions of total human capital invested in research and stock of technology.

The scale effect describes the situation that a country's per capita economic growth rate is relatively higher when its inputs of production, such as human capital or the labor force, increase. Accordingly, a larger country, in terms of labor or human capital, must grow faster than a small country. In Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992), the innovation rate depends upon the amount of resources devoted into R&D. As the population or the aggregate human capital stock grows at a constant rate, the economy would grow at an exponential rate. Therefore, the scale effect does exist in these models. However, Jones (1995b) found that over the past 4 decades or so, the education level, the quality and the number of researchers, and the amount of resources invested in R&D increased significantly, yet the income per capita data showed no upward trend for OECD countries. There should be no scale effect.

Since Jones (1995b), there has been considerable amount of efforts in attempting to negate the scale effect. Jones (1999), Dinopoulos and Thompson (1999) and Dinopoulos and Sener (2007) classified these papers into several groups.

The first group that fought this scale effect includes Jones (1995a), Kortum (1997), Segerstrom (1998) and Li (2003). Jones (1995a) built his model on the Romer (1990) model; Kortum (1997) set up the model to incorporate the concepts of Pareto efficient re-

²A relaxation of this assumption will not alter the conclusion too much.

search distribution and technology frontier; Segerstrom (1998) constructed his model with vertical innovation along with the semi-conductor industry example and Li (2003) further expanded this model. Although the structures of these models are different, they shared the same characteristic – the diminishing technological opportunity. Jones (1995a) argued, “Perhaps the most obvious ideas are discovered first so that the probability that a person engaged in R&D discovers a new idea is decreasing in the level of knowledge”. So, this group of models targeted directly on the technology production function. They believed that the reason why we will have the scale effect in Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992) is simply because the technology production functions in these models are assumed linear. A linear technology production function represents constant returns to scale in inputs. As the population or the human capital stock increases, more technologies are produced. A non-linear technology production function representing diminishing technological opportunity could prevent the scale effect from happening.

The second group, including Aghion and Howitt (1998), Dinopoulos and Thompson (1998), Young (1998), and Peretto (1998), took another path – the variety expansion. They argued that as the population grows both the market and the labor force become larger. The number of researchers in a single industry should be larger if the number of total industries is fixed. But if one permits the number of total industries to expand while the number of researchers in each industry just stays the same as the total population grows, then there should be no scale effect in the economy. So, this group of endogenous growth models relies on the linear relationship between the level of total population and the number of the variety of products to remove the scale effect.

Jones (1999) showed that if one relaxes this linear relationship then the models in this group will yield the same results either as Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) with the scale effect, or as Jones (1995a), Kortum (1997), Segerstrom (1998), and Li (2003) without the scale effect and welfare implications. Hence, he thought that the linear restriction is too rigid. However, Dinopoulos and Thompson

(1999) counter-argued that this linearity is derived from a solid microeconomic foundation and hence the results should be acceptable.

The third group of works asserts that the economy is deviating from its balanced growth path (BGP). As the economy is on its BGP, the long-run economic growth rate should be constant over time. Most development economists believe that a constant growth rate observed from the U.S. GDP per capita data suggests to us that the economy is on its BGP. However, Jones (2002) pointed out another possibility – the economy is off the BGP. If it is off the BGP, as the resources devoted to the R&D sector (including the increase in the population, human capital and the education level) keep increasing, the economy could grow at a constant rate that is higher than the one on the BGP. And Jones believes that this could explain why we don't observe the scale effect.

All the works we have seen above can be categorized as the endogenous technological progress growth models. There is another parallel to the endogenous growth literature – the endogenous human capital growth literature. In a seminal paper, Lucas (1988) presented an optimal control model with human capital accumulation in explaining long-run economic growth. In this model, a general production function is used to produce a single good. The inputs are physical capital and human capital. All individuals are identical. Each individual chooses the allocation of human capital between education and working at each point in time. As he allocates more human capital into education in any point of time, he accumulates more human capital. He will have more total human capital stock at the next time point. But this also means that he allocates less human capital in working and thus earns less at that point of time. He faces a trade off between a higher earning this point in time and next point in time. He needs to find a balanced time path of the allocation of his human capital. In this model, the famous Uzawa-Lucas human capital production function or the human capital accumulation behavior is

$$\dot{h}_t = \delta\mu_t h_t$$

where h_t denotes total human capital stock accumulated at t , \dot{h}_t denotes instant change

of total human capital stock at t , δ denotes a parameter, μ_t denotes the fraction of time allocating to education at t .

This function shows that as an individual allocates more of his human capital at t , $h_t\mu_t$, into education, he will accumulate more human capital, \dot{h}_t . Put in another words, as he devotes higher fraction of time into education effort, μ_t , he will accumulate human capital, $\frac{\dot{h}_t}{h_t}$, faster. As the parameter δ is replaced by δ^γ , where the power of δ ($\gamma < 1$) representing diminishing education returns, human capital growth rate $\frac{\dot{h}_t}{h_t}$ falls to zero eventually. It is impossible to have a sustained economic growth. But when γ is larger than 1, the economy will explode eventually. Further, a common observation tells us that an individual accumulates human capital “rapidly early in life, then less rapidly, then not at all – as though each additional percentage increment were harder to gain than the preceding one” (Lucas, 1988). But if the life span is extended to infinite horizon, an individual should not face this problem. Hence, following Uzawa (1965), it is suitable to assume that γ equals to 1.

To close the model, Lucas used a normal budget constraint. Hence, in this optimization problem, each individual chooses consumption and the allocation of human capital to maximize his own lifetime utility. Two state variables are human capital and physical capital. By solving this present value Hamiltonian, Lucas found that the optimal fraction of human capital allocated to education is constant. This leads to a constant human capital accumulation rate, a constant physical capital accumulation rate, and a constant economic growth rate. This constant economic growth rate is independent of the scale of the economy.

Recently, these two parallel lines of thought intertwine. A few works have integrated the endogenous technological growth models and the endogenous human capital accumulation growth models. Arnold (1998) used Grossman and Helpman (1991) as a foundation and replaced population growth with human capital accumulation. He followed Lucas (1988) and assumed that each identical individual chooses consumption and the allocation of human capital between education and work to maximize his own lifetime utility. He is also subject to his lifetime budget constraint and human capital accumulation constraint. In Lucas

(1988), an economic agent puts the effort into education and accumulates human capital to increase the final good productivity and the wage rate. Aronld argued that an economic agent accumulates human capital to increase the productivity of R&D. This leads to an increased innovation rate and hence total productivity and the wage rate. So, human capital accumulation is the true engine of economic growth. His results are similar to those found in Lucas (1988). Because each individual's optimal rate of human capital accumulation is constant in the model, the innovation rate and the economic growth rate are constant.

Arnold (2002) applied the same idea and replaced Grossman and Helpman (1991) by Segerstrom (1998), a vertical innovation model. Strulik (2005) used Li (2000), which contains both horizontal and vertical innovations and spillovers, to incorporate human capital accumulation in the model. However, the results are closer to Li (2000) in that policy is irrelevant to economic growth. Papageorgiou and Perez-Sebastian (2006) constructed their model based on Jones (1995a) in which innovation and imitation co-exist and endogenous human capital formation through formal education explains the Japanese and the South Korean development experiences successfully. With high human capital level and low physical capital level after W.W.II, Japan had a high innovation rate at the beginning and gradually slowed down to the normal growth rate. On the other hand, with a low human capital stock after W.W.II, South Korean had a high imitation rate and then gradually reduced to the normal growth rate. But again, no policies can promote long run growth rate in this model.

In general, the main idea of this group of works is to show that human capital accumulation provides the fuel for R&D activities and economic growth. All of them succeeded in explaining this main idea. However, they all adopted the Uzawa-Lucas type of human capital accumulation behavior. This type of accumulation function provides little information on how to create new knowledge. All one knows is that if an agent puts more effort into education, he will accumulate human capital faster. In the first model of the dissertation, economic agents are assumed to be motivated by monopoly rents and create new knowledge to expand the knowledge base from R&D activities and then learn from this

knowledge base. Because it is harder for economic agents to fully understand and handle a knowledge that is closer to the knowledge frontier, it is harder for economic agents to learn. Most importantly, this type of accumulation function still leads us to the scale effect. The human capital accumulation and economic growth rates depend on the education productivity. Since, the education productivity have increased over time for countries in O.E.C.D., the human capital accumulation and economic growth rates should grow over time. Hence, there is the scale effect in these models. Again, we do not observe this situation from empirical studies.

2.3 Reverse Brain Drain Literature

Return migration of skilled laborers has been recognized as one of the sources that could lead to a brain gain effect³. Several well-known works are worth noting.

Borjas and Bratsberg (1996) constructed a model to allow for individual self-selection. Their finding is that those average skilled workers have a tendency of return migration. As the economic environments of the sending country improve over time, they would find it appealing for them to migrate back to their country. Hence, it is the average skilled labors that return to the sending country. As they return to the sending country, the overall human capital stock expands and a brain gain effect occurs.

Stark et al. (1997) presented a model that allows for heterogeneous ability individuals. All individuals are allowed to choose the level of human capital accumulation. Under these assumptions, relatively lower ability individuals could acquire more human capital and migrate to the receiving country. Over time, employers discover individual abilities and adjust the wages according to their productivities. Those immigrants with relatively lower abilities face lower wages and decide to return to the sending country. This leads to a brain gain effect for the sending country.

In these two works, we have only seen those migrants with relatively lower abilities

³The other two sources are remittances and networks.

return to the sending country. In reality, return migrants could be skilled laborers and they could bring the new human capital acquired in the receiving country back to the sending country. Especially in the study of the reversed brain drain, one can expect this situation to occur. However, one can't find this argument in these two works.

In terms of the economic incentives that attract migrants back to the sending country, Borjas and Bratsberg (1996) and Mayr and Peri (2008) paid their attentions to the wage premium. Wage premium serves as the instrument in attracting brains back to their motherland. In this paper, motivated by cases in Taiwan and India as mentioned above, we argue that it is the opportunities to succeed together with the wage premium that completes the full circle of brain drain and reverse brain drain circulation.

2.4 Summary of the Literature Review

This chapter begins by reviewing related brain drain literature. From the brain drain literature, we have seen the historical evolution of attitudes toward skilled international migrants. Within the perfect competition setting, skilled emigrants have no impacts on the overall human capital accumulation. The introduction of market imperfections leads us to a much more realistic environment and result – the brain drain effect. However, emigration could also lead to a brain gain effect. The uncertainty about the emigration outcome and the return migration are the main sources of the brain gain effect. As long as the brain gain effect dominates the brain drain effect, the overall human capital stock in the sending country is higher. The sending country could benefit from skilled labor outflows.

Despite the complexity and variability in the outcomes and the underlying assumptions, further research can be conducted on several related topics and help create a more comprehensive body of brain drain literature.

First, previous works in brain drain literature highly rely on the tie between human capital accumulation and economic growth. They show us that skilled labor out-migration might lead to an increased overall human capital level for the remaining individuals in the

sending country, and this increased overall human capital stock could lead to an economic growth. The endogenous growth literature showed us that both innovations and human capital accumulation are the driving forces for long-term economic growth. However, there is only one true engine of economic growth – human capital accumulation. An integrated innovation and human capital accumulation model can explain this idea well. But the Uzawa-Lucas human capital accumulation constraint in this type of models requires some modifications, including the explanation of the source of human capital stock for economic agents to learn from and the difficulty of learning. An integrated model can also eliminate the scale effect. In the chapter 3, an endogenous technological progress growth model with a modified human capital accumulation constraint is designed to serve this purpose.

Second, several works in the reverse brain drain literature refer wage premium as the sole source of economic incentive that attracts skilled migrants returning to the sending country. Inspired by Taiwanese experience, the opportunity to succeed in the sending country is also considered a major economic incentive for an emigrant to return. In chapter 4, the second model is constructed in a self-selection framework, which is commonly used in the brain drain literature, to examine this new theory.

3 THE FIRST MODEL

As mentioned above, this chapter develops an integrated innovations and human capital accumulation growth model with new learning behavior. Along the way, a benchmark model is presented in parallel to other integrated growth models. Next, this benchmark model is further modified to incorporate a new learning behavior. Finally, international migration is discussed within the context of this modified model.

The foundation of the first model is based on Barro and Sala-i-Martin's model and this model contains the scale effect in Romer's fashion. We take two steps to negate the scale effect. First, following Arnold (1998), we incorporate the Uzawa-Lucas type of human capital accumulation behavior which is commonly used in the endogenous human capital growth literature. i.e. human capital accumulation rate equals to education productivity times the education effort. Arnold claimed that in this way the human capital accumulation rate is determined by the education productivity on balanced growth path (because education effort stays the same over time on the balanced growth path) and thus human capital grows at a constant rate. Because the human capital available in the economy grows at a constant rate, the amount of human capital devoted to R&D grows at the same rate. The innovation rate grows at the rate of human capital accumulation. Finally, the economy grows at a rate that equals to the innovation rate and is independent of the scale of human capital.

However, education productivities in O.E.C.D. countries have increased significantly over the past two decades. According to Arnold, we should be able to observe an economy in O.E.C.D. grows at a higher rate. Once again, we don't observe this phenomenon. So, in the second step, we introduce the ability to learn (which is inherent) in replacing

the education productivity (that improves over time) and each individual learns from the knowledge base created through innovations. i.e. $\dot{h}_t = \varphi\mu_t(\bar{h}_t - h_t)$. Ability to learn is commonly used in the brain drain literature. So, this set up not only negates the scale effect but also connects the endogenous growth literature and the brain drain literature. The result of our modified model shows that the ability to learn determines the human capital growth rate (unlike Arnold's education productivity), the innovation rate, and consequently, the economic growth rate (like Arnold (1998)).

3.1 A benchmark model

This section presents a benchmark model. Its features are similar to Barro and Sala-i-Martin (2004) vertical innovation type endogenous technological progress growth model, except that here I follow Arnold (1998) in integrating the Uzawa-Lucas human capital accumulation behavior.

3.1.1 Descriptions of the benchmark model

There are four types of economic agents in this economy: final good producers, R&D producers, intermediate goods producers, and individuals. All economic agents behave rationally. The economy is endowed with a fixed input, such as land.

3.1.1.1 Final Good Producers A homogenous final good is produced under perfect competition. Each of the final good producers faces a same production technology, and employs a fraction of the fixed factor F and intermediate goods. The final good is chosen as numeraire, hence it is sold at unit price to individuals. This Cobb-Douglas production function is written as

$$Y_i = F_i^{1-\alpha} \sum_{j=1}^N \left(q^{k_j} x_{ij} \right)^\alpha \quad (3.1)$$

where $i = 1, 2, 3, \dots, M$ denotes the i^{th} final good producer and M is a large number, $j = 1, 2, 3, \dots, N$ denotes the j^{th} intermediate good producer, Y_i denotes the output for the final good producer i , F_i denotes the amount of the fixed factor F employed by firm i , x_{ij} denotes the amount of the intermediate good j employed by firm i , q^{kj} denotes the quality of intermediate good j , and $0 < \alpha < 1$ is a parameter.

The original quality of a product is normalized to one. Each new improvement pushes the quality up by q . So, q^{kj} means the firm j currently produces its intermediate good at the k rung on the ladder⁴. Because of higher productivity, a final good producer prefers a higher quality intermediate good than the same intermediate good but with lower quality. In this way, a higher quality product makes a lower quality one obsolete, and the firm that improves the quality takes over the market.

This production function exhibits constant returns to scale in F_i and $q^{kj} x_{ij}$ together. Further, there are neither substitutions nor complements among intermediate goods. This specification is important to this model because it implies that a new quality improved intermediate good will not make other intermediate goods obsolete. The marginal product of x_{ij} , $MP_{x_{ij}}$, is written as $\alpha F_i^{1-\alpha} (q^{kj})^\alpha (x_{ij})^{\alpha-1}$. $MP_{x_{ij}}$ must be the same across all j . Otherwise, firm i will use more intermediate good j with higher $MP_{x_{ij}}$ than another intermediate good j' but with lower $MP_{x_{ij'}}$. Therefore, α should be the same for all j .

From $p_j = \frac{w}{\alpha B}$ (3.8), the prices of j are the same regardless of its quality. Also, the marginal product of the intermediate good j with quality q^{kj} is higher than the marginal product of the intermediate good j with q^{kj-1} . Accordingly, the final good producer i will employ the intermediate good j at the highest quality available. It is in this way that a higher quality intermediate good j makes a lower quality one, j' , obsolete, and the firm that improves the quality over previous quality takes over the market. Hence, only those intermediate goods with highest qualities available enter equation (3.1).

A final good producer takes both the prices of F and intermediate goods as given and

⁴Each intermediate good is produced by a single producer. See 3.1.1.2 for more information.

seeks its own profit maximization. The profit of each final good producer is

$$\pi_i = F_i^{1-\alpha} \sum_{j=1}^N \left(q^{kj} x_{ij} \right)^\alpha - \sum_{j=1}^N p_j x_{ij} \quad (3.2)$$

where π_i denotes profit of firm i , and p_j denotes the price of the intermediate good j

The first order condition is:

$$\frac{\partial \pi_i}{\partial x_{ij}} = \alpha F_i^{1-\alpha} \left(q^{kj} \right)^\alpha \left(x_{ij} \right)^{\alpha-1} - p_j = 0 \quad (3.3)$$

From (3.3),

$$x_{ij} = F_i \left[\frac{\alpha (q^{kj})^\alpha}{p_j} \right]^{\frac{1}{1-\alpha}} \quad (3.4)$$

is the optimal usage of intermediate good j used by final good producer i .

Define x_j as the aggregate intermediate good j used by all i final good producers:

$$x_j \equiv \sum_i x_{ij} = \sum_i F_i \left[\frac{\alpha (q^{kj})^\alpha}{p_j} \right]^{\frac{1}{1-\alpha}} = F \left[\frac{\alpha (q^{kj})^\alpha}{p_j} \right]^{\frac{1}{1-\alpha}} \quad (3.5)$$

Where $F \equiv \sum_i F_i$

3.1.1.2 Intermediate Goods Producers An R&D firm that succeeds in improving the quality of a specific intermediate good obtains a permanent patent and becomes the only producer of this good. In this economy, there are N intermediate goods⁵ in total and the production technology is the same for all N intermediate goods producers. Human capital from the individuals is the sole input in production process.

Because of the patent law, an intermediate good market is monopolized. A rational intermediate good producer sets the price of its intermediate good to maximize its own profit given the wage rate, w . With the assumption that each unit of human capital is used

⁵ N is assumed large enough for us to guarantee that a solution of $\frac{\partial}{\partial}$ does exist.

in producing B unit of x_j , the instantaneous monopoly profit flow for an intermediate good j with quality q^{k_j} is

$$\pi(k_j) = p_j x_j - w H_j \quad (3.6)$$

Bring x_j from (3.5) into (3.6),

$$\pi(k_j) = \left(p_j - \frac{w}{B}\right) F \left[\frac{\alpha (q^{k_j})^\alpha}{p_j} \right]^{\frac{1}{1-\alpha}} \quad (3.7)$$

Take the derivative with respect to p_j ,

$$\frac{\partial \pi(k_j)}{\partial p_j} = \frac{\alpha}{1-\alpha} p_j^{\frac{1}{\alpha-1}} F \alpha^{\frac{1}{1-\alpha}} (q^{k_j})^{\frac{\alpha}{1-\alpha}} - \frac{w}{B} \left(\frac{1}{\alpha-1} \right) p_j^{\frac{\alpha-2}{1-\alpha}} F \alpha^{\frac{1}{1-\alpha}} (q^{k_j})^{\frac{\alpha}{1-\alpha}} = 0$$

$$p_j = \frac{w}{\alpha B} \quad (3.8)$$

w is constant over time⁶ and α and B are two parameters. Hence, p_j is also constant over time and identical for all intermediate goods. This is because the cost of production of each intermediate good j is the same and each good j enters into the final good production function 3.1 symmetrically.

Bring p_j from (3.8) back to (3.5), $x_j = F \left[\frac{\alpha (q^{k_j})^\alpha}{\frac{w}{\alpha B}} \right]^{\frac{1}{1-\alpha}}$.

Define new variables: $X \equiv \sum_{j=1}^N x_j$ denotes the total quantity of intermediate goods from 1 to N , and, $H_x = \sum_{j=1}^N H_j$ denotes the total amount of human capital employed in intermediate goods production process in the economy. Also, $Q \equiv \sum_{j=1}^N (q^{k_j})^{\frac{\alpha}{1-\alpha}}$ denotes the sum of total

⁶This can be seen in (3.39).

quality of intermediate goods. Thus,

$$X = F\alpha^{\frac{2}{1-\alpha}}B^{\frac{1}{1-\alpha}}w^{\frac{1}{\alpha-1}}Q \quad (3.9)$$

and

$$H_x = \frac{X}{B} = F\alpha^{\frac{2}{1-\alpha}}B^{\frac{\alpha}{1-\alpha}}w^{\frac{1}{\alpha-1}}Q \quad (3.10)$$

Bring (3.9) in (3.1), the aggregate final good produced becomes

$$Y \equiv \sum_i Y_i = F^{1-\alpha} \sum_{j=1}^N \left(q^{k_j}\right)^\alpha (x_j)^\alpha = F\alpha^{\frac{2\alpha}{1-\alpha}}B^{\frac{\alpha}{1-\alpha}}w^{\frac{\alpha}{\alpha-1}}Q \quad (3.11)$$

Consequently, (3.6) becomes

$$\pi(k_j) = (1-\alpha)F\alpha^{\frac{1+\alpha}{1-\alpha}}B^{\frac{\alpha}{1-\alpha}}w^{\frac{\alpha}{\alpha-1}}\left(q^{k_j}\right)^{\frac{\alpha}{1-\alpha}} \quad (3.12)$$

Although the patent obtained is permanent, the monopoly rent is temporary. The total profit flow that an intermediate good producer earns starts from the time it improves the quality at t_{k_j} up to the time its product is replaced by another firm with a higher quality at $t_{k_{j+1}}$. After being replaced by another firm, the patent still exists but the monopoly rent for this firm drops to zero. The present value of this profit flow is written as

$$V(k_j) = \int_{t=t_{k_j}}^{t_{k_{j+1}}} \pi(k_j) e^{-\bar{r}(v,t_{k_j}) \bullet (v-t_{k_j})} dv \quad (3.13)$$

where $\bar{r} \equiv \left(\frac{1}{v-t_{k_j}}\right) \int_{t_{k_j}}^v r(\omega) d\omega$ is the average interest rate, representing the average opportunity cost over this time period.

Because interest rate is constant over time⁷ on the balance growth path, (3.13) can be simplified as:

$$V(k_j) = \pi(k_j) \left[1 - e^{-r(t_{k_{j+1}}-t_{k_j})}\right] / r$$

⁷This can be seen in (3.37) below.

Let the probability per unit of time of a successful innovation in sector j with the quality q^{k_j} be $Pa(k_j)$. Put in another words, $Pa(k_j)$ is the probability per unit of time that an outside firm raises the quality from q^{k_j} to $q^{k_{j+1}}$ in sector j . Accordingly, the expected value of profit flow is

$$E[V(k_j)] = \frac{\pi(k_j)}{r + Pa(k_j)} \quad (3.14)$$

3.1.1.3 R&D Activity The probability to success, $p_a(k_j)$, depends on the total R&D effort in sector j . Let $H_j^{R\&D}$ be the aggregate flow of human capital devoted by all potential innovators in sector j when the leading quality is q^{k_j} . A higher level of $H_j^{R\&D}$ leads to a higher probability of success, $p_a(k_j)$ ⁸. The probability to success also depends on q^{k_j} . Following Barro and Sala-i-Martin (2004), the probability is assumed to be negatively related to the output that would be produced at the improved quality level. The higher the targeted quality⁹ is, the harder it is to success. Hence, the probability to success is

$$p_a(k_j) = \frac{H_j^{R\&D}}{\xi(q^{k_{j+1}})^{\frac{\alpha}{1-\alpha}}} \quad (3.15)$$

where $\xi > 0$ is a constant.

Because of free entry in R&D market, a firm can choose whether it wants to invest in an R&D activity to improve the quality of an existing intermediate good or not. It would invest in R&D activity if the benefit of this activity can at least cover the cost. The benefit of this activity is $p_a(k_j) E[V(k_{j+1})]$, and the cost is $wH_j^{R\&D}$. Hence, a firm would invest if $p_a(k_j) E[V(k_{j+1})] - wH_j^{R\&D} \geq 0$. However, the R&D market is perfectly competitive. A firm can enter this market without being obstructed. If there is profit in the market, the firm will want to enter this market. This arbitrage behavior¹⁰ ends when $p_a(k_j) E[V(k_{j+1})] - wH_j^{R\&D} = 0$.

⁸A more realistic diminishing returns assumption will remove the scale effect but will not alter the basic result. Please see Jones (1995b), Kortum (1997), Segerstrom (1998), and Li (2003).

⁹ $(q^{k_{j+1}})^{\frac{\alpha}{1-\alpha}}$

¹⁰A firm can choose to improve one intermediate good's quality or the others.

Replacing $p_a(k_j)$ from (3.15) and $E[V(k_{j+1})]$ from (3.14),

$$\frac{H_j^{R\&D}}{\xi(q^{k_{j+1}})^{\frac{\alpha}{1-\alpha}}} \frac{(1-\alpha)F\alpha^{\frac{1+\alpha}{1-\alpha}}w^{\frac{\alpha}{\alpha-1}}(q^{k_{j+1}})^{\frac{\alpha}{1-\alpha}}}{r+p_a(k_{j+1})} - wH_j^{R\&D} = 0$$

$$r+p_a(k_{j+1}) = (1-\alpha)F\alpha^{\frac{1+\alpha}{1-\alpha}}B^{\frac{\alpha}{1-\alpha}}w^{\frac{1}{\alpha-1}}\xi^{-1} \quad (3.16)$$

Because r is constant on the balance growth path and w is also a constant, is a constant overtime and across j .

To obtain the optimal level of $H_j^{R\&D}$, one can bring (3.15) into (3.16).

$$r + \frac{H_j^{R\&D}}{\xi(q^{k_{j+1}})^{\frac{\alpha}{1-\alpha}}} = (1-\alpha)F\alpha^{\frac{1+\alpha}{1-\alpha}}B^{\frac{\alpha}{1-\alpha}}w^{\frac{1}{\alpha-1}}\xi^{-1}$$

Thus, the optimal level of $H_j^{R\&D}$ is:

$$H_j^{R\&D} = \left[(1-\alpha)F\alpha^{\frac{1+\alpha}{1-\alpha}}B^{\frac{\alpha}{1-\alpha}}w^{\frac{1}{\alpha-1}} - r\xi \right] (q^{k_{j+1}})^{\frac{\alpha}{1-\alpha}} \quad (3.17)$$

The aggregate flow of human capital devoted to R&D is:

$$H^{R\&D} \equiv \sum_{j=1}^N H_j^{R\&D} = \left[(1-\alpha)F\alpha^{\frac{1+\alpha}{1-\alpha}}B^{\frac{\alpha}{1-\alpha}}w^{\frac{1}{\alpha-1}} - r\xi \right] q^{\frac{\alpha}{1-\alpha}} Q \quad (3.18)$$

and the total amount of resource devoted to R&D is:

$$Z \equiv wH^{R\&D} = w \left[(1-\alpha)F\alpha^{\frac{1+\alpha}{1-\alpha}}B^{\frac{\alpha}{1-\alpha}}w^{\frac{1}{\alpha-1}} - r\xi \right] q^{\frac{\alpha}{1-\alpha}} Q \quad (3.19)$$

The total amount of resource devoted to R&D is the total amount of human capital devoted to R&D multiplies by the wage rate.

3.1.1.4 Individuals There are a fixed number of forever-living¹¹ individuals in the economy. These individuals are identical in every respect except the ability to learn. The distribution of the ability to learn is uniform. A representative individual is the individual with the weighted average of the ability to learn of all individuals. Hence, his economic behavior times the total number of individuals is equal to the aggregate behavior. To make it tractable, in the benchmark model, a typical Uzawa-Lucas human capital accumulation behavior is used. This behavior simply specifies how an individual learns new knowledge. It does not tell us where the knowledge comes from for this individual to learn.

An individual's instantaneous utility function takes the form of constant relative risk aversion utility $u_t(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta}$ where c_t denotes the consumption at time t and $\theta > 0$ determines the household's willingness to shift consumption between different periods. This functional form is needed for the economy to converge to a balanced growth path. The coefficient of relative risk aversion is defined as $-c_t \frac{u''(c_t)}{u'(c_t)}$ and equals to θ in this case. Since there is no uncertainty in this model, the household's attitude toward risk is not relevant. But θ also determines the household's willingness to shift consumption between different periods. When θ is smaller, marginal utility falls more slowly as consumption rises, and so the household is more willing to allow its consumption to vary over time.

The goal of this representative individual is to maximize the present value of his lifetime utility $U = \int_{t=0}^{\infty} u_t(c_t) e^{-\rho t} dt$ or

$$U = \int_{t=0}^{\infty} \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \quad (3.20)$$

subject to the budget constraint,

$$\dot{a}_t = w_t(1 - \mu_t)h_t + ra_t - c_t \quad (3.21)$$

and the human capital formation constraint,

¹¹The forever-living assumption allows us to avoid having to deal with the problem of ability distributions across generations.

$$\dot{h}_t = \varphi \mu_t h_t \quad (3.22)$$

where U denotes the lifetime utility level of this representative individual, $\rho > 0$ denotes the rate of time preference, a_t denotes the asset holding at t , $0 \leq \mu_t \leq 1$ denotes the fraction of time spent in the learning activity (the learning effort) instead of working at t , h_t denotes the amount of human capital accumulated up to t , φ denotes the ability to learn of this representative individual, and a variable with a dot on top denotes the derivative of this variable with respect to t .

Time preference, ρ , is different from the coefficient of the relative risk aversion, θ . This individual discounts the future instantaneous utility at the rate of time preference. i.e. utility received later has less value to him. θ can be obtained from the marginal rate of substitution ($\frac{1}{\theta}$) of the instantaneous utility function $u_t(c_t) = \frac{c_t^{1-\theta}-1}{1-\theta}$. However, ρ and θ are related to each other. An individual places more weights on his current consumption than his future consumptions when ρ or θ is higher.

An individual holds assets in the form of financial assets (savings and dissavings). He accumulates financial assets from working and from financial assets return. He can also borrow from other individuals to smooth his lifetime consumption. However, he can't borrow more than his lifetime assets to prevent Ponsi game. Here, we do not specify the type of financial asset market. It could be a competitive market in which that the interest rate or the rate of return of the financial assets, r , is equal to the time preference, ρ , or it could be an imperfectly competitive market in which r is different than ρ .

Notice that the parameter φ denotes the ability to learn. In the growth modeling, one would assume that it is the same for all individuals. However, it is common to assume that φ varies across individuals in the brain drain literature. In order to blend a brain drain model into a growth model, φ is assumed to be heterogeneous across individuals. The

Hamiltonian equation is written as

$$J_1 = \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \lambda_1 [w_t (1 - \mu_t) h_t + r a_t - c_t] + \lambda_2 [\phi \mu_t h_t] \quad (3.23)$$

The control variables are c_t and μ_t , and the state variables are a_t and h_t .

3.1.2 Equilibrium and BGP Analysis

From (3.23), the first order conditions for the Hamiltonian equation are

$$\frac{\partial J_1}{\partial c_t} = c_t^{-\theta} e^{-\rho t} - \lambda_1 = 0 \quad (3.24)$$

$$\frac{\partial J_1}{\partial \mu_t} = -\lambda_1 w_t h_t + \lambda_2 \phi h_t = 0 \quad (3.25)$$

$$\frac{\partial J_1}{\partial a_t} = \lambda_1 r = -\dot{\lambda}_1 \quad (3.26)$$

$$\frac{\partial J_1}{\partial h_t} = \lambda_1 w_t (1 - \mu_t) + \lambda_2 \phi \mu_t = -\dot{\lambda}_2 \quad (3.27)$$

(3.24) is equivalent to $c_t^{-\theta} e^{-\rho t} = \lambda_1$

Total differentiate w.r.t. t ,

$$-\theta c_t^{-\theta-1} \dot{c}_t e^{-\rho t} - c_t^{-\theta} \rho e^{-\rho t} = \dot{\lambda}_1 \quad (3.28)$$

For the optimal consumption path, we can bring (3.24) and (3.28) into (3.71).

$$c_t^{-\theta} e^{-\rho t} r = \theta c_t^{-\theta-1} \dot{c}_t e^{-\rho t} + \rho c_t^{-\theta} e^{-\rho t}$$

$$\frac{\dot{c}_t}{c_t} = \frac{r - \rho}{\theta} \quad (3.29)$$

This condition is a classic result. It states that individual's consumption is rising if the rate of return exceeds the rate of discounting. A higher rate of return means higher future income and thus more resources for consumption. A lower discounting rate means less income is discounted over time. Recall that a smaller θ means an individual is more willing to allow variations in his consumption pattern over time. Consequently, individual's consumption is more responsive to the difference between the rate of return and the rate of discounting.

Analogously, for the optimal path of wage rate, we can total differentiate (3.25) w.r.t. t .

$$\dot{\lambda}_2 = [-\lambda_1 r w_t + \lambda_1 \dot{w}_t] / \varphi \quad (3.30)$$

Bring (3.25) and (3.27) into (3.30),

$$\lambda_1 w_t (1 - \mu_t) + \lambda_1 w_t \mu_t = \frac{\lambda_1 r w_t - \lambda_1 \dot{w}_t}{\varphi}$$

Because $h \neq 0$,

$$\frac{\dot{w}_t}{w_t} = r - \varphi \quad (3.31)$$

This equation states that wage rate has to rise sufficiently fast to induce human capital investment. The higher interest rate it is and the lower the learning ability it is, the larger the required growth rate of real wage rate. If the left-hand side of (3.31) were larger than the right-hand side, an individual would over-invest in education. If the reverse happens, an individual would over-invest into financial market and under-invest in education.

To yield the optimal path of total quality Q ,

$$E \left(\frac{\Delta Q}{Q} \right) = p_a \left[\sum_{j=1}^N \left(q^{k_{j+1}} \right)^{\frac{\alpha}{1-\alpha}} - \sum_{j=1}^N \left(q^{k_j} \right)^{\frac{\alpha}{1-\alpha}} \right] / Q = p_a Q \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) / Q$$

Because N is large, one can apply the law of large numbers to seek for the value as Δ is approaching 0,

$$\frac{\dot{Q}}{Q} = \left[(1 - \alpha) F \alpha^{\frac{1+\alpha}{1-\alpha}} B^{\frac{\alpha}{1-\alpha}} w^{\frac{1}{\alpha-1}} \xi^{-1} - r \right] \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \quad (3.32)$$

The sum of total quality rises as fixed factor F , B , and real wage w are larger and/or the rate of return r and the opportunity parameter ξ are smaller. The more fixed factor is devoted into production, more output is produced. More profit is created from an intermediate good improvement. A higher real wage rate means a higher payment for the service of research. A higher B means a relatively higher payoff for each unit of human capital used in R&D, and a smaller rate of return means a smaller opportunity cost of engaging in a R&D activity. Further, a smaller opportunity parameter, ξ , means it is easier to succeed in R&D. A new improved intermediate good is replaced by another one sooner. All of these make researchers more willing to improve the quality of products. Consequently, total quality of intermediate goods expands faster.

Because all others are constant, (3.32) shows that $\frac{\dot{Q}}{Q}$ is a linear function of r . To obtain the growth rate of aggregate final good, Y , one can total differentiate (3.11) w.r.t. t . The result is

$$\frac{\dot{Y}}{Y} = \frac{\alpha}{1-\alpha} \frac{\dot{w}_t}{w_t} + \frac{\dot{Q}}{Q} \quad (3.33)$$

By totally differentiating (3.9) w.r.t. t , we find

$$\frac{\dot{X}}{X} = \frac{1}{\alpha-1} \frac{\dot{w}_t}{w_t} + \frac{\dot{Q}}{Q} \quad (3.34)$$

The aggregate final good Y is used in producing intermediate goods, consumption goods, improving qualities of intermediate goods, and education. The aggregate level of resource constraint can be written as

$$Y = X + C + Z + E \quad (3.35)$$

where E denotes the total resources used in learning.

Because (3.35) is a linear function, the growth rates of all variables are the same. Define γ as the growth rate of these variables. Finally, we establish

$$\gamma = \frac{\dot{Y}}{Y} = \frac{\dot{X}}{X} = \frac{\dot{C}}{C} = \frac{\dot{Z}}{Z} = \frac{\dot{E}}{E}$$

Because the growth rate of Y is the same as the growth rate of X , we can equalize (3.33) and (3.34). Therefore,

$$\frac{\alpha}{\alpha - 1} \frac{\dot{w}_t}{w_t} + \frac{\dot{Q}}{Q} = \frac{1}{\alpha - 1} \frac{\dot{w}_t}{w_t} + \frac{\dot{Q}}{Q}$$

Hence,

$$\frac{\dot{w}_t}{w_t} = 0 \quad (3.36)$$

The growth rate of wage rate is constant on BGP. Recall that the quality of an intermediate good improved is set to q , which is a constant, all the innovators and all the intermediate good producers are the same in terms of innovation technologies and production technologies. This means that the expected values of profit flow of improvements are the same. Also, recall that human capital is the only input in R&D and intermediate good production. Consequently, real wage rate per unit of human capital should remain constant over time.

Wage rate is constant and all variables in (3.35) are functions of Q . As a result,

$$\gamma = \frac{\dot{Y}}{Y} = \frac{\dot{X}}{X} = \frac{\dot{C}}{C} = \frac{\dot{Z}}{Z} = \frac{\dot{E}}{E} = \frac{\dot{Q}}{Q}$$

From this and from (3.29) and (3.32), one gets

$$\left[(1 - \alpha) F \alpha^{\frac{1+\alpha}{1-\alpha}} B^{\frac{\alpha}{1-\alpha}} w^{\frac{1}{\alpha-1}} \xi^{-1} - r \right] \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) = \frac{r - \rho}{\theta}$$

With some manipulations, one can get interest rate, r .

$$r = \frac{(1 - \alpha) F \alpha^{\frac{1+\alpha}{1-\alpha}} B^{\frac{\alpha}{1-\alpha}} w^{\frac{1}{\alpha-1}} \xi^{-1} \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta + \rho}{1 + \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta} \quad (3.37)$$

Also, from (3.31) and (3.36), $\frac{\dot{w}_t}{w_t} = r - \varphi = 0$. We have

$$r = \varphi \quad (3.38)$$

So, r is constant on BGP and equals to φ , which means that the rate of return is equal to the representative individual's ability. Because we assume that education effort remains unchanged, human capital grows at the rate of φ . Productivity grows at φ . The opportunity cost of not investing in education is thus φ . (3.38) leads us to the wage rate, w . By equating (3.37) and (3.38), φ becomes

$$\varphi = \frac{(1 - \alpha) F \alpha^{\frac{1+\alpha}{1-\alpha}} B^{\frac{\alpha}{1-\alpha}} w^{\frac{1}{\alpha-1}} \xi^{-1} \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta + \rho}{1 + \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta}$$

So,

$$w = \left[\frac{\varphi + \varphi \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta - \rho}{(1 - \alpha) F \alpha^{\frac{1+\alpha}{1-\alpha}} B^{\frac{\alpha}{1-\alpha}} \xi^{-1} \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta} \right]^{(\alpha-1)} \quad (3.39)$$

This condition confirms that real wage rate is constant over time.

Because the number of individuals is constant, the aggregate consumption growth rate is equal to the representative individual's consumption growth rate. Clearly, with (3.38), we can rewrite (3.29) as

$$\gamma = \frac{\dot{C}}{C} = \frac{\dot{c}}{c} = \frac{r - \rho}{\theta} = \frac{\varphi - \rho}{\theta} \quad (3.40)$$

This equality shows that economic growth rate is constant over time, is positively re-

lated to the ability to learn, and is negatively related to the rate of time preference, ρ , and the individual's willingness to shift consumption between different period, θ .

To solve for the rest of the variables, we can bring the wage rate, w , from (3.39) back to (3.18), (3.9), (3.10), (3.11), and (3.16) respectively.

From (3.18) & (3.39),

$$H^{R\&D} = \left[(1 - \alpha) F \alpha^{\frac{1+\alpha}{1-\alpha}} B^{\frac{\alpha}{1-\alpha}} w^{\frac{1}{\alpha-1}} - r\xi \right] q^{\frac{\alpha}{1-\alpha}} Q = \xi \left[\frac{\varphi - \rho}{\left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta} \right] q^{\frac{\alpha}{1-\alpha}} Q \quad (3.41)$$

As the economy is on its BGP, t will allocate more human capital to R&D activities when it is harder to improve the quality of an intermediate good, i.e. φ is larger and when the growth rate of the economy γ is larger.

From (3.9) & (3.39),

$$X = \left\{ \frac{\alpha B \xi \left[\varphi + \varphi \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta - \rho \right]}{(1 - \alpha) \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta} \right\} Q \quad (3.42)$$

Recall that $p_j = \frac{w}{\alpha B}$. As B becomes larger, an intermediate good price p becomes lower. This would increase the demand for this intermediate good. Also, recall that all intermediate goods are charged at the same price. As B becomes larger, aggregate intermediate goods expand. The learning ability and the discount rate are also factors that determine X . A higher learning ability for the representative individual and a lower discount rate mean a higher economic growth rate. Consequently, more X is needed in the final good production.

Since one unit of human capital can be used in producing B units of an intermediate good (3.10), we can re-arrange (3.39) as

$$H_x = \frac{X}{B} = \left\{ \frac{\alpha \xi \left[\varphi + \varphi \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta - \rho \right]}{(1 - \alpha) \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta} \right\} Q \quad (3.43)$$

This gives the aggregate amount of human capital required in all intermediate goods

production.

Substituting wage rate from (3.39) into (3.11), we have the final good production as

$$Y = \left\{ \frac{\xi^\alpha \alpha^\alpha B^\alpha \left[\varphi + \varphi \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta - \rho \right] \alpha}{(1-\alpha)^\alpha F^{\alpha-1} \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right)^\alpha \theta^\alpha} \right\} Q \quad (3.44)$$

With the wage rate (3.39), one can further simplify P_a in (3.16) as

$$p_a = \frac{\varphi - \rho}{\left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta} \quad (3.45)$$

i.e., p_a is proportional to the economic growth rate $\frac{(\varphi-\rho)}{\theta}$. Furthermore, it is constant as stated before. The representative individual's learning ability determines the probability to success. This probability to success then determines the economic growth rate.

3.1.3 Properties of the benchmark model

Because all variables in (3.35) are linear functions of Q , this benchmark model contains only one state variable, Q . Given an initial value of Q , all of Y , X , Z , E , and C are growing at the growth rate of Q , with $\gamma = \frac{\varphi-\rho}{\theta}$ (3.40). There are no transitional dynamics. Further, an increased human capital level will not raise the economic growth rate. Therefore, this benchmark model negates the scale effect¹².

In this benchmark model as well as other integrated growth models, the main argument is that human capital is the true engine of economic growth. Human capital growth leads to a growth of R&D outlay, Q . Since the learning effort is constant on BGP, growth rate of Q should be constant over time. This can be seen as the reason in eliminating the scale effect.

However, this benchmark model is different from others in that a vertical R&D growth model is used in replacing a horizontal R&D growth model. In the integrated growth

¹²Other integrated models use the Uzawa-Lucas type of human capital accumulation. Hence, as the education productivity improves over time, the scale effect shows up. However, unlike others, the economic growth rate depends on the ability to learn which is a constant in the benchmark model. This assures that there is no scale effect when education productivity increases over time.

literature, one cannot find an integrated model built on top of a vertical R&D model. The alternation in this benchmark model is aimed to fill the gap left by the integrated growth literature.

At the first sight, this conclusion seems odd. We build this benchmark model to show that human capital is the true engine of economic growth. Then we find that human capital level will not alter the economic growth rate. However, this benchmark model is the perfect candidate to defend this argument. The reason why the scale effect is removed in this benchmark model is that we assume that each individual lives forever. Under this assumption, the representative individual will devote the same amount of learning efforts over time. In overlapping-generation models, representative individuals from different generations devotes the same amount of learning efforts. Since the amount of learning efforts is the same across time both in forever living and overlapping-generation models, human capital grows at the rate of learning ability (3.22). And learning ability determines the rate of technological progress (3.40) and thus the rate of economic growth. We can restate our conclusion as following,

When each individual makes the same learning effort over time, learning ability determines the rate of economic growth. And when each individual devotes different learning effort over time, growth rate of human capital (learning effort multiplies the ability) determines the rate of economic growth.

3.2 A Modified Model

Up to this point the analysis has been focused on a typical Uzawa-Lucas human capital accumulation behavior, in which individuals learn from a knowledge base growing exogenously. More realistically, however, knowledge should come from R&D activities. This chapter presents a model with a modified human capital accumulation behavior to address this issue.

3.2.1 Descriptions of the Model

As the benchmark model, there are four types of economic agents in this economy: final good producers, R&D firms, intermediate goods producers, and individuals. All agents behave rationally. This modified model differs from the benchmark model only on “individual behavior”. Therefore, we skip final goods producers’ section, intermediate goods producers’ section, and R&D activity’s section, and directly go into the description of individual behavior to highlight the essence of this modified model.

The goal of the representative individual is to maximize

$$U = \int_{t=0}^{\infty} \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \quad (3.46)$$

subjects to the budget constraint

$$\dot{a}_t = w_t (1 - \mu_t) h_t + r a_t - c_t \quad (3.47)$$

and the modified human capital accumulation constraint

$$\dot{h}_t = \varphi \mu_t (\bar{h}_t - h_t) \quad (3.48)$$

where h_t denotes the amount of human capital accumulated up to t , $\bar{h}_t \equiv \tau Q$ denotes the maximum amount of human capital at t , $\tau > 0$ is a parameter, φ denotes the ability to learn for an individual, μ_t denotes the effort of an individual at t .

This new human capital accumulation behavior illuminates the idea that an individual acquires the knowledge from innovations. It also accounts for the situation that it is harder for an individual to acquire a knowledge that is closer to the technology frontier as represented by \bar{h}_t . As the knowledge is closer to the technology frontier, human capital grows more slowly with the same amount of learning efforts, μ_t . The representative individual takes w_t , r , and \bar{h}_t as given, and chooses μ_t and c_t to maximize his lifetime utility. The

Hamiltonian equation is written as

$$J_2 = \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \lambda_1 [w_t(1-\mu_t)h_t + ra_t - c_t] + \lambda_2 [\varphi\mu_t(\bar{h}_t - h_t)] \quad (3.49)$$

The control variables are still c_t and μ_t , and the state variables are a_t and h_t .

3.2.2 General Equilibrium

First order conditions of (3.49) are,

$$\frac{\partial J_2}{\partial c_t} = c_t^{-\theta} e^{-\rho t} - \lambda_1 = 0 \quad (3.50)$$

$$\frac{\partial J_2}{\partial \mu_t} = -\lambda_1 w_t h_t + \lambda_2 \varphi (\bar{h}_t - h_t) = 0 \quad (3.51)$$

$$\frac{\partial J_2}{\partial a_t} = \lambda_1 r = -\dot{\lambda}_1 \quad (3.52)$$

$$\frac{\partial J_2}{\partial h_t} = \lambda_1 w_t (1 - \mu_t) - \lambda_2 \varphi \mu_t = -\dot{\lambda}_2 \quad (3.53)$$

Following the same process as in the benchmark model, we can derive the growth rates of c and w . First, let us total differentiate (3.50) w.r.t. t . This gives

$$-\theta c_t^{-\theta-1} \dot{c}_t e^{-\rho t} - c_t^{-\theta} \rho e^{-\rho t} = \dot{\lambda}_1 \quad (3.54)$$

Second, let us bring (3.50) and (3.54) into (3.52). We find

$$c_t^{-\theta} e^{-\rho t} r = \theta c_t^{-\theta-1} \dot{c}_t e^{-\rho t} + \rho c_t^{-\theta} e^{-\rho t}$$

That is

$$\frac{\dot{c}_t}{c_t} = \frac{r - \rho}{\theta} \quad (3.55)$$

To obtain the growth rate of w , we can total differentiate (3.51) w.r.t. t ,

$$\dot{\lambda}_2 = \frac{-\lambda_1 r w_t h_t + \lambda_1 \dot{w}_t h_t + \lambda_1 w_t \dot{h}_t - \lambda_1 w_t h_t \frac{\dot{\bar{h}}_t - \dot{h}_t}{\bar{h}_t - h_t}}{\varphi(\bar{h}_t - h_t)} \quad (3.56)$$

Then, we bring (3.51) & (3.56) into (3.53). The growth rate of wage rate is

$$\frac{\dot{w}_t}{w_t} = r + \frac{\dot{\bar{h}}_t}{\bar{h}_t - h_t} - \varphi \frac{\bar{h}_t - h_t}{h_t} \quad (3.57)$$

The expected growth rate of Q is

$$E\left(\frac{\Delta Q}{Q}\right) = p_a \left[\sum_{j=1}^N \left(q^{k_{j+1}}\right)^{\frac{\alpha}{1-\alpha}} - \sum_{j=1}^N \left(q^{k_j}\right)^{\frac{\alpha}{1-\alpha}} \right] / Q = p_a Q \left(q^{\frac{\alpha}{1-\alpha}} - 1\right) / Q$$

The variety of intermediate goods, N , is assumed to be large. By the law of large numbers, the expected growth rate of Q is approaching $\frac{\dot{Q}}{Q}$ as $\Delta \rightarrow 0$. Therefore, the growth rate of Q is

$$\frac{\dot{Q}}{Q} = \left[(1-\alpha) F \alpha^{\frac{1+\alpha}{1-\alpha}} B^{\frac{\alpha}{1-\alpha}} \xi^{-1} - r \right] \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \quad (3.58)$$

From (3.11), the growth rate of Y is

$$\frac{\dot{Y}}{Y} = \frac{\alpha}{1-\alpha} \frac{\dot{w}_t}{w_t} + \frac{\dot{Q}}{Q} \quad (3.59)$$

From (3.9), the growth rate of X is

$$\frac{\dot{X}}{X} = \frac{1}{\alpha-1} \frac{\dot{w}_t}{w_t} + \frac{\dot{Q}}{Q} \quad (3.60)$$

The aggregate level resource constraint is

$$Y = X + C + Z + E \quad (3.61)$$

E denotes the aggregate amount of education efforts.

Because (3.61) is linear, all variables should grow at the same rate.

$$\gamma = \frac{\dot{Y}}{Y} = \frac{\dot{X}}{X} = \frac{\dot{C}}{C} = \frac{\dot{Z}}{Z} = \frac{\dot{E}}{E}$$

The growth rate of final good, intermediate good, consumption, the aggregate effort in R&D, and the aggregate effort in education should be the same. This can be seen in this way: If the aggregate education effort grows at the rate γ , the human capital grows at γ . The amount of human capital available for R&D and intermediate goods production should grow at γ . Consequently, the growth rate of the aggregate quality, and the wage per person should grow at γ . Consumption and thus the final good production also grow at γ .

By equating (3.59) and (3.60), we have

$$\frac{\alpha}{\alpha - 1} \frac{\dot{w}_t}{w_t} + \frac{\dot{Q}}{Q} = \frac{1}{\alpha - 1} \frac{\dot{w}_t}{w_t} + \frac{\dot{Q}}{Q}$$

Now, we can restate the growth rate of wage as

$$\frac{\dot{w}_t}{w_t} = 0 \tag{3.62}$$

This growth rate is the same as the one in the benchmark model. It is constant on BGP. Recall that the quality of an improved intermediate good remains q , and innovation technologies and production technologies are the same across intermediate goods. These allow the same expected values of profit flows of improvements. Under the assumption that human capital is the only input in R&D and intermediate goods production, real wage rate per unit of human capital should remain constant over time.

Because all variables in (3.61) are linear functions of Q and the growth rate of w is zero, the following equality holds.

$$\frac{\dot{Q}}{Q} = \frac{\dot{C}}{C}$$

As a result, from (3.55) & (3.58), we have

$$\left[(1 - \alpha) F \alpha^{\frac{1+\alpha}{1-\alpha}} B^{\frac{\alpha}{1-\alpha}} w^{\frac{1}{\alpha-1}} \xi^{-1} - r \right] \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) = \frac{r - \rho}{\theta}$$

Hence,

$$r = \frac{(1 - \alpha) F \alpha^{\frac{1+\alpha}{1-\alpha}} B^{\frac{\alpha}{1-\alpha}} w^{\frac{1}{\alpha-1}} \xi^{-1} \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta + \rho}{1 + \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta} \quad (3.63)$$

(3.57) & (3.62) lead us to

$$\frac{\dot{w}_t}{w_t} = r + \frac{\dot{\bar{h}}_t}{\bar{h}_t - h_t} - \varphi \frac{\bar{h}_t - h_t}{h_t} = 0$$

That is

$$r + \frac{\dot{\bar{h}}_t}{\bar{h}_t - h_t} - \varphi \frac{\bar{h}_t - h_t}{h_t} = 0$$

The representative individual chooses h_t while taking \bar{h}_t as given. We can obtain the optimal human capital

$$h_t = \frac{\left(r \bar{h}_t + \dot{\bar{h}}_t + 2\varphi \bar{h}_t \right) \pm \sqrt{\left(r \bar{h}_t + \dot{\bar{h}}_t + 2\varphi \bar{h}_t \right)^2 - 4(\varphi + r)(\varphi \bar{h}_t^2)}}{2(\varphi + r)}$$

$$\therefore \bar{h}_t = \tau Q$$

$$\therefore \dot{\bar{h}}_t = \tau \dot{Q} = \tau \left(\frac{\dot{Q}}{Q} \right) Q = \tau \left(\frac{r - \rho}{\theta} \right) Q$$

We can rewrite the equation as

$$h_t = \frac{[\theta r + (r - \rho) + 2\theta\varphi] \frac{\bar{h}_t}{\theta} \pm \sqrt{[\theta r + (r - \rho)]^2 + 4\theta\varphi(r - \rho)} \frac{\bar{h}_t}{\theta}}{2(\varphi + r)}$$

with only the negative term for h_t ¹³ being chosen. Because $[\theta r + (r - \rho)]^2 + 4\theta\varphi(r - \rho)$ is larger than 0, h_t is not a complex number. Since the right-hand-side of this equation is a constant multiplied by \bar{h}_t on BGP, h_t equals a constant term times \bar{h}_t . The implication is that the representative individual always learns the same portion of the maximum amount of human capital. Due to the difficulty of learning, he will seek for knowledge that is easier to acquire. Because a harder one will become easier over time, he will wait until the harder one becomes easier. Hence, he learns the knowledge from the easiest up to a certain level of difficulty according to his learning ability.

$$\text{Set } h_t = \frac{1}{\delta+1} \bar{h}_t$$

where δ is a constant, $0 \leq \frac{1}{1+\delta} \leq 1$ and $\delta \geq 0$ or $\delta \leq -1$ (Pick $\delta \geq 0$ because an individual should acquire a positive amount of human capital)

Consequently, (3.48) can now be expressed as

$$\dot{h}_t = \varphi\mu_t (\bar{h}_t - h_t) = \varphi\mu_t \delta h_t \quad (3.64)$$

Hence, an individual chooses the amount of efforts equal to $\mu_t \delta$. Recall that the amount of efforts in the benchmark model is μ_t . So, when it is harder for an individual to learn newer technology, this individual needs to put more efforts in learning in order to maximize his lifetime utility.

A representative individual maximizes

$$U = \int_{t=0}^{\infty} \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \quad (3.65)$$

subject to the budget constraint

¹³Because this positive term makes $h_t > \bar{h}_t$ as $\varphi = r$.

$$\dot{a}_t = w_t(1 - \delta\mu_t)h_t + ra_t - c_t \quad (3.66)$$

and the human capital formation constraint

$$\dot{h}_t = \varphi\delta\mu_t h_t \quad (3.67)$$

The Hamiltonian equation becomes

$$J_3 = \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \lambda_{1\delta} [w_t(1 - \delta\mu_t)h_t + ra_t - c_t] + \lambda_{2\delta} [\varphi\delta\mu_t h_t] \quad (3.68)$$

Compared to the Hamiltonian in the benchmark model (3.23), notice that μ_t has been modified by δ in this new Hamiltonian.

First order conditions of this new Hamiltonian equation are,

$$\frac{\partial J_3}{\partial c_t} = c_t^{-\theta} e^{-\rho t} - \lambda_{1\delta} = 0 \quad (3.69)$$

$$\frac{\partial J_3}{\partial \mu_t} = -\lambda_{1\delta} w_t \delta + \lambda_{2\delta} \varphi \delta = 0 \quad (3.70)$$

$$\frac{\partial J_3}{\partial a_t} = \lambda_{1\delta} r = -\dot{\lambda}_{1\delta} \quad (3.71)$$

$$\frac{\partial J_3}{\partial h_t} = \lambda_{1\delta} w_t (1 - \delta\mu_t) + \lambda_{2\delta} \varphi \delta \mu_t = -\dot{\lambda}_{2\delta} \quad (3.72)$$

These first order conditions are similar to the ones obtained in the benchmark model from (3.24) to (3.27). Following the procedures in the benchmark model, we can find the growth rates of c and w from these first order conditions. To derive the growth rate of c , we can total differentiate (3.69) w.r.t. t ,

$$-\theta c_t^{-\theta-1} \dot{c}_t e^{-\rho t} - c_t^{-\theta} \rho e^{-\rho t} = \dot{\lambda}_{1\delta} \quad (3.73)$$

Bring (3.69) and (3.73) into (3.71),

$$c_t^{-\theta} e^{-\rho t} r = \theta c_t^{-\theta-1} \dot{c}_t e^{-\rho t} + \rho c_t^{-\theta} e^{-\rho t}$$

That is

$$\frac{\dot{c}_t}{c_t} = \frac{r - \rho}{\theta} \quad (3.74)$$

which shows that the consumption path is the same as the one in the benchmark model.

To derive the growth rate of w , we, then, total differentiate (3.70) w.r.t. t .

$$\dot{\lambda}_{1\delta} w_t + \lambda_{1\delta} \dot{w}_t = \dot{\lambda}_{2\delta} \varphi$$

$$\dot{\lambda}_{2\delta} = [-\lambda_{1\delta} r w_t + \lambda_{1\delta} \dot{w}_t] / \varphi \quad (3.75)$$

From (3.70), (3.72) and (3.75),

$$\lambda_{1\delta} w_t (1 - \delta \mu_t) + \lambda_{1\delta} w_t \delta \mu_t = \frac{\lambda_{1\delta} r w_t - \lambda_{1\delta} \dot{w}_t}{\varphi}$$

Because $h \neq 0$,

$$\frac{\dot{w}_t}{w_t} = r - \varphi \quad (3.76)$$

which is also identical to (3.31) in the benchmark model.

(3.31) and (3.62) together imply that

$$\frac{\dot{w}_t}{w_t} = r - \varphi = 0$$

So,

$$r = \varphi \quad (3.77)$$

When the economy is on its balanced growth path the rate of return on assets, r , equals to the ability to learn, φ . An individual chooses whether to invest in the asset market or in education. Only when the rate of return from assets equals to the rate of return from education, an individual does not favor one over another. Recall that the ability to learn is inherently given. From $r = \frac{(1-\alpha)F\alpha^{\frac{1+\alpha}{1-\alpha}}B^{\frac{\alpha}{1-\alpha}}w^{\frac{1}{\alpha-1}}\xi^{-1}(q^{\frac{\alpha}{1-\alpha}}-1)\theta+\rho}{1+(q^{\frac{\alpha}{1-\alpha}}-1)\theta}$ and

$w = \left[\frac{\varphi+\varphi(q^{\frac{\alpha}{1-\alpha}}-1)\theta-\rho}{(1-\alpha)F\alpha^{\frac{1+\alpha}{1-\alpha}}B^{\frac{\alpha}{1-\alpha}}\xi^{-1}(q^{\frac{\alpha}{1-\alpha}}-1)\theta} \right]^{(\alpha-1)}$, we know that as φ increases or decreases, w adjusts and then r adjusts accordingly to match new φ .

From (3.63) and (3.77), φ can also be expressed as

$$\varphi = \frac{(1-\alpha)F\alpha^{\frac{1+\alpha}{1-\alpha}}B^{\frac{\alpha}{1-\alpha}}w^{\frac{1}{\alpha-1}}\xi^{-1}(q^{\frac{\alpha}{1-\alpha}}-1)\theta+\rho}{1+(q^{\frac{\alpha}{1-\alpha}}-1)\theta}$$

which yield the expression for the wage rate:

$$w = \left[\frac{\varphi+\varphi(q^{\frac{\alpha}{1-\alpha}}-1)\theta-\rho}{(1-\alpha)F\alpha^{\frac{1+\alpha}{1-\alpha}}B^{\frac{\alpha}{1-\alpha}}\xi^{-1}(q^{\frac{\alpha}{1-\alpha}}-1)\theta} \right]^{(\alpha-1)} \quad (3.78)$$

From (3.74) and (3.77), the economic growth rate can be expressed as:

$$\gamma = \frac{\dot{C}}{C} = \frac{\dot{c}}{c} = \frac{r-\rho}{\theta} = \frac{\varphi-\rho}{\theta} \quad (3.79)$$

This growth rate is the same as the one in the benchmark model, but with richer interpretation: Recall that Y , X , C , Z , and E are growing at the same rate γ . (3.79) shows that growth rates of these variables are constant over time, are positively related to φ , and are negatively related to ρ and θ . A higher ability to learn means that the representative individual learns more, and more human capital can be used in R&D. This should lead to a higher wage rate and income per person and more demand on the final good. More demand means more final good production and higher economic growth rate. A higher discount rate means more production is being discounted. This reduces the economic growth rate.

Finally, a smaller θ means this representative individual is less willing to save the current income for future consumption. A higher current consumption means higher current final good production and economic growth.

Bring (3.78) into (3.18), (3.9), (3.10), (3.11), and (3.16), we reach the solutions for $H^{R\&D}$, X , H_x , Y , and P_a , respectively.

$$H^{R\&D} = \left[(1 - \alpha) F \alpha^{\frac{1+\alpha}{1-\alpha}} B^{\frac{\alpha}{1-\alpha}} w^{\frac{1}{\alpha-1}} - r \xi \right] q^{\frac{\alpha}{1-\alpha}} Q = \xi \left[\frac{\varphi - \rho}{\left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta} \right] q^{\frac{\alpha}{1-\alpha}} Q \quad (3.80)$$

$$X = \left\{ \frac{\alpha B \xi \left[\varphi + \varphi \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta - \rho \right]}{(1 - \alpha) \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta} \right\} Q \quad (3.81)$$

$$H_x = \frac{X}{B} = \left\{ \frac{\alpha \xi \left[\varphi + \varphi \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta - \rho \right]}{(1 - \alpha) \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta} \right\} Q \quad (3.82)$$

$$Y = \left\{ \frac{\xi^\alpha \alpha^\alpha B^\alpha \left[\varphi + \varphi \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta - \rho \right] \alpha}{(1 - \alpha)^\alpha F^{\alpha-1} \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right)^\alpha \theta^\alpha} \right\} Q \quad (3.83)$$

$$p_a = \frac{\varphi - \rho}{\left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \theta} \quad (3.84)$$

Once again, p_a is constant.

The aggregate human capital stock condition requires that

$$(1 - \mu_t) H_t = H_x + H^{R\&D}$$

Because μ_t is constant on BGP, the growth rate of H_t is equal to the growth rates of H_x

and $H^{R\&D}$, i.e., $\frac{\dot{H}_t}{H_t} = \frac{\dot{H}_x}{H_x} = \frac{\dot{H}^{R\&D}}{H^{R\&D}}$. From (3.80) and (3.82), $\frac{\dot{H}_x}{H_x} = \frac{\dot{H}^{R\&D}}{H^{R\&D}} = \frac{\dot{Q}}{Q}$. So,

$$\frac{\dot{H}_t}{H_t} = \frac{\dot{Q}}{Q} \quad (3.85)$$

The aggregate quality grows at the human capital growth rate. This condition confirms that human capital is the true engine of the economy.

3.2.3 Properties of the Modified Model

Once again, this model contains only one state variable, Q . All variables in (3.61) grow at the same rate as Q at $\frac{\varphi-\rho}{\theta}$, which is the same as the one obtained in the benchmark model. Further, no transitional dynamics exist. Since the human capital level does not affect the economic growth rate, this model also shows no scale effect.

Although the basic results of this model are similar to the ones in the benchmark model, the key concepts are different. In the benchmark model as well as other integrated growth models, the human capital creation process is just like a black box. All we know is that if you put more human capital into this box, you will get more human capital. It cannot explain what creates the knowledge for us to learn from. Here, in this modified model, as motivated by monopoly profit a firm devotes its resources in conducting R&D. As this firm succeeds in improving the quality of an intermediate good, it also creates a byproduct, knowledge. It is by this mechanism that knowledge base expands for individuals to learn from.

3.2.4 Skilled Labor Migration

First, let us further expand this model to a two-country model: a sending country and a receiving country. In the initial state, two countries are identical in every respect, except that the receiving country has higher technology, Q , than the sending country.

Recall that this model does not present any scale effect. In this case, it is impossible for

both countries to have higher growth rates by accumulating more human capital. However, for the receiving country, if it “imports” high learning ability individuals from the sending country, the learning ability of the representative individual, φ , increases. From (3.79) and (3.85), an increase in φ leads to an increase in the human capital accumulation rate and thus the economic growth rate, γ .

However, the problem for the sending country is much more complicated. Because of higher wages compared to the ones in the sending country, high-learning-ability individuals would want to leave the sending country and enter the labor market in the receiving country. The representative individual’s φ decreases in the sending country after out-migration of skilled laborers. From (3.79) and (3.85), a decrease in φ leads to a decrease in the human capital accumulation rate and thus the economic growth rate, γ . A brain drain effect occurs.

But it is not the end of the story. If one introduces the uncertainty about migration, one could obtain a brain gain effect as well. Let us introduce a new variable b_t , such that $b_t w_t = (1 - P_{migration}) w_{sending,t} + P_{migration} w_{receiving,t}$ is the expected wage rate for the representative individual that has the migration opportunity, where $P_{migration}$ is the probability of migration, $w_{sending,t}$ is the wage rate in the sending country, and $w_{receiving,t}$ is the wage rate in the receiving country at time t . The budget constraint (3.66) becomes

$$\dot{a}_t = b_t w_t (1 - \delta \mu_t) h_t + r a_t - c_t \quad (3.86)$$

and the new Hamiltonian equation is

$$J_4 = \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \lambda_{1\delta} [b_t w_t (1 - \delta \mu_t) h_t + r a_t - c_t] + \lambda_{2\delta} [\varphi \delta \mu_t h_t] \quad (3.87)$$

Following the same method to solve for the growth rate of wage rate, we obtain

$$\frac{\dot{w}_t}{w_t} + \frac{\dot{b}_t}{b_t} = r - \varphi \quad (3.88)$$

From (3.62) and (3.88), we obtain

$$\frac{\dot{b}_t}{b_t} = r - \varphi \quad (3.89)$$

So, (3.79) can be re-written as

$$\gamma = \frac{\dot{H}_t}{H_t} = \frac{\dot{Q}}{Q} = \frac{\dot{C}}{C} = \frac{\dot{c}}{c} = \frac{r - \rho}{\theta} = \frac{\frac{\dot{b}_t}{b_t} + \varphi - \rho}{\theta} \quad (3.90)$$

The impact of out-migration on the growth rate of human capital accumulation and the economic growth rate depends on the term $\frac{\dot{b}_t}{b_t} + \varphi$. Out-migration leads to an increase in φ and thus w for the receiving country, and a decrease in φ and thus w for the sending country. An increase in the difference between w for the receiving country and the sending country leads to an increase in $\frac{\dot{b}_t}{b_t}$ and thus an increase in human capital accumulation rate and economic growth rate, γ . This is the brain gain effect. If the brain gain effect could dominate the brain drain effect, the sending country could enjoy a higher human capital accumulation rate and economic growth rate.

3.2.5 Skilled Labor Return Migration and Social Planner's Problem

If the sending country could provide some economic incentives, such as wage premium, which is a subsidy on the wage rate, and the opportunity to succeed to attract those skilled laborers back to the sending country, a return-migration flow occurs. The complete mechanism of pulling back is constructed in the next chapter. Due to the limit of this model, we focus on the impacts of returnees upon the economic growth of the sending country in this section.

(3.79) states that as the ability of the representative individual of a country improves, the economic growth rate of this country rises. Skilled laborer's out-migration tends to reduce φ for the sending country while increasing φ for the receiving country. As mentioned above, the impact of out-migration on the economic growth of the sending country depends on the values of (3.90) before and after the migration occurs. The impact on the economic growth

on the receiving country is certain. During the period that the migrants stay, the receiving country enjoys a higher growth rate and this raises per capita output level permanently¹⁴.

Return-migration contributes to the economy of the sending country in two ways. First, the economy of the sending country grows faster permanently. After pulling back skilled migrants, ϕ is reduced and thus the economic growth rate, Υ , for the receiving country. At the same time, this return-migration flow would increase ϕ and γ for the sending country. Second, income per capita rises in the sending country. The technology that returnees bring back from the receiving country contributes to the stock of the human capital of the sending country for an individual to learn. The level of Q increases permanently, and the growth rate of Q rises temporary until the economy reaches its new balanced growth path. Y is a linear function of Q . Income per capita is also a linear function of Q . Consequently, the level of income per capita increases permanently, and the growth rate of income per capita rises temporary.

¹⁴Because the number of individuals in both countries is constant excepting migration and return-migration, per capita output moves the same direction as total output.

4 THE SECOND MODEL

The model in this chapter follows a common self-selection fashion. Each individual optimally chooses his human capital level and the location he works. The new elements in this model, as opposed to others, are that each individual chooses the amount of higher education and that each migrant faces two economic incentives, namely, the wage premium and the opportunity to succeed in the sending country, that induce his returning.

Consider a small open economy, S , with overlapping generations of individuals who live for two periods: working and retirement. Time goes from 0 to infinite. In the first period, an agent supplies all his human capital for production and allocates a share of his wage earning to education. In the second period, he relies only on the interest earned from saving while working. Heterogeneity is introduced in the sense that each agent exhibits different levels of ability to learn. Inter-generational transmission of human capital promotes economic growth.

4.1 Production Sector

Production of the consumption good is carried out by a single representative firm operating under the Cobb-Douglas technology:

$$Y_t = F(K_t, H_t) = K_t^\alpha H_t^{1-\alpha} \quad (4.1)$$

where K_t denotes aggregate physical capital at time t , H_t aggregate human capital, Y_t aggregate quantity of the consumption good produced, and $\alpha \in (0, 1)$. Physical capital

depreciates in one period completely. The intensive form of this production function: $y_t = f(k_t)$ where $y_t = Y_t/H_t$ and $k_t = K_t/H_t$. We make the standard assumptions about the function f , namely $f(k_t) > 0$, $f'(k_t) > 0$, $f''(k_t) < 0$, $\forall k_t > 0$ and the Inada conditions $\lim_{k \rightarrow 0} f(k_t) = 0$, $\lim_{k \rightarrow 0} f'(k_t) = \infty$, $\lim_{k \rightarrow \infty} f'(k_t) = 0$. The rate of return to physical capital or the interest rate r_t is equal to $r_t = f'(k_t) = \alpha k_t^{\alpha-1}$. The world is in a steady state equilibrium. Thus, the world's interest rate r_t^* is constant. Because the economy is small and the mobility of physical capital is perfect, domestic interest rate r_t is also fixed at r_t^* . This further determines the level of k_t and the wage rate per effective worker $w_t = f(k_t) - k_t f'(k_t) = (1 - \alpha) k_t^\alpha$. Hence, we will write $r_t = r$ and $w_t = w$, $\forall t$.

4.2 Individual Behavior

Each individual, i , cares about his first period consumption c_{1t} and second period consumption c_{2t+1} . With perfect foresight, he has to choose the education level e_t^i , and the consumption distribution between c_{1t} and c_{2t+1} so as to maximize lifetime utility which is defined as:

$$u_t^i = \ln c_{1t}^i + \frac{1}{1+\rho} \ln c_{2t+1}^i \quad (4.2)$$

where time preference is $\rho < 1$. His human capital level depends on the average human capital level of last generation $t-1$ \bar{h}_t , his education level e_t^i , and his ability to learn a^i :

$$h_t^i = a^i (e_t^i)^\beta \bar{h}_t \quad (4.3)$$

Following de la Croix and Doepke (2003) and Chen (2009), the parameter $\beta \in (0, 1)$ captures the education productivity and \bar{h}_t represents the average human capital level of teachers. Following Mountford (1997) and Beine et al. (2001), we consider a uniform distribution of abilities to learn: $a^i \in [\underline{a}, \bar{a}]$.

Education here is defined as advanced education such as higher education. Because education is costly, some individuals may find themselves better off without acquiring ed-

ucation. For an individual without education, his budget constraint is:

$$w\bar{h}_t = c_{1t} + \frac{1}{1+r}c_{2t+1} \quad (4.4)$$

Because we examine the effect of brain drain activity on S, education is required for the emigration from S to R. In order to motivate the desire for emigration, we impose a requirement that the wage rate per effective labor in R, w^* , to be equal to ηw where $\eta > 1$. The probability of a successful emigration $p \in [0, 1]$ is assumed to be the same for all educated individuals. When facing p , a risk-neutral individual has the opportunity to earn a higher wage rate in R. Hence, for an individual with education, his budget constraint is:

$$(1 - p + p\eta) wa (e_t^i)^\beta \bar{h}_t = c_{1t} + \frac{1}{1+r}c_{2t+1} + e_t^i w \bar{h}_t \quad (4.5)$$

where $e_t^i w \bar{h}_t$ represents the education cost.

Although w^* is higher than w , with proper incentives, some successful emigrants still want to return to S. The incentives include wage premium over those stayers and domestic investment opportunities. The budget constraint for a returnee is:

$$[1 - p + p\mu\eta + p(1 - \mu)X] wa (e_t^i)^\beta \bar{h}_t = c_{1t} + \frac{1}{1+r}c_{2t+1} + e_t^i w \bar{h}_t \quad (4.6)$$

where $\mu \in [0, 1]$ denotes the fraction of working time spending in R, and $X \geq 0$ is the payoff after returning to S.

As Mayr and Peri (2008) stated, some emigrants might want to return to S because of two reasons - wage premium and domestic investment opportunity. Some countries would place a premium on skilled returnees as monetary incentives to attract the return of valuable technology and management skills. A good example is that China recently offers up to 100% wage subsidies for engineers in some specific fields. Also, as S climbs up the development ladder, new profitable investment opportunities might surface. Examples are

Taiwan and India. When Taiwan wanted to upgrade its industries from traditional manufacturing to I.T., Taiwanese government built a science park to enhance human capital spillover and offered financial and management supports to reinforce investment opportunities. Emigrants have the advantages in the newly emerged investment opportunities over their non-emigrants counterparts. They simply bring back what they have learned in R to have a head start ahead of the stayers. Mayr and Peri (2008) combine these two reasons and simply argue that returnees accumulate some human capital during their stay in R and thus they have the premium in S. This paper differs from Mayr and Peri (2008) in this respect. We argue that both wage premium and the domestic investment opportunities could lead to the accumulation of the overall human capital stock in S, but the latter is more powerful. We will return to this point later in the calibration section and simply use X to represent both for now.

An individual's problem is to choose c_{1t} , c_{2t+1} , e_t^i so as to maximize his lifetime utility subject to his budget constraint. For an individual choosing not to acquire advance education, his first period consumption and second period consumption are written as:

$$c_{1t} = \frac{1 + \rho}{2 + \rho} w \bar{h}_t \quad (4.7)$$

and

$$c_{2t+1} = \frac{1 + r}{2 + \rho} w \bar{h}_t \quad (4.8)$$

For a potential migrant, his e_t^i , c_{1t} and c_{2t+1} are:

$$e_t^i = (1 - p + p\eta)^{\frac{1}{1-\beta}} a^{i\frac{1}{1-\beta}} \beta^{\frac{1}{1-\beta}} \quad (4.9)$$

$$c_{1t} = \frac{1 + \rho}{2 + \rho} (1 - p + p\eta)^{\frac{1}{1-\beta}} a^{i\frac{1}{1-\beta}} \left(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}} \right) w \bar{h}_t \quad (4.10)$$

$$c_{2t+1} = \frac{1 + r}{2 + \rho} (1 - p + p\eta)^{\frac{1}{1-\beta}} a^{i\frac{1}{1-\beta}} \left(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}} \right) w \bar{h}_t \quad (4.11)$$

For a potential returnee, his e_t^i , c_{1t} and c_{2t+1} are:

$$e_t^i = [1 - p + p\mu\eta + p(1 - \mu)X]^{\frac{1}{1-\beta}} a^{i\frac{1}{1-\beta}} \beta^{\frac{1}{1-\beta}} \quad (4.12)$$

$$c_{1t} = \frac{1 + \rho}{2 + \rho} [1 - p + p\mu\eta + p(1 - \mu)X]^{\frac{1}{1-\beta}} a^{i\frac{1}{1-\beta}} \left(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}} \right) w\bar{h}_t \quad (4.13)$$

$$c_{2t+1} = \frac{1 + r}{2 + \rho} [1 - p + p\mu\eta + p(1 - \mu)X]^{\frac{1}{1-\beta}} a^{i\frac{1}{1-\beta}} \left(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}} \right) w\bar{h}_t \quad (4.14)$$

In all three cases, the individual distributes all his lifetime income into consumption. The distribution depends upon the time discount factor ρ and interest rate r . Note that when the probability p is zero (i.e. Closed to emigration), both equation 4.9 and 4.12 collapse to $e_t^i = a^{i\frac{1}{1-\beta}} \beta^{\frac{1}{1-\beta}}$. That is the amount of education an individual would like to acquire without emigration. It is clear that, whether this individual is a potential migrant or returnee, he would make more education effort when p is not 0. Further, a potential returnee would make more effort than a potential migrant would do. A potential returnee also accumulates more human capital than a potential migrant does. Return migration creates another channel of brain gain.

Taking derivatives of equation 4.9 and 4.12 with respect to p , we obtain

$$\frac{\partial e^i}{\partial p} = a^{(1/(1-\beta))} \beta^{(1/(1-\beta))} \frac{1}{1-\beta} (\eta - 1)^{(\beta/(1-\beta))} > 0 \quad (4.15)$$

and

$$\frac{\partial e^i}{\partial p} = a^{(1/(1-\beta))} \beta^{(1/(1-\beta))} \left(\frac{1}{1-\beta} \right) [-1 + \mu\eta + (1 - \mu)X]^{\beta/(1-\beta)} > 0 \quad (4.16)$$

Hence, when facing an increase in the probability of migration, both a potential emigrant and a potential returnee invest more into education.

For an individual to choose migration, his expected lifetime income as a potential emi-

grant must be equal to or higher than his expected income as a stayer. i.e.

$$(1 - p + p\eta) wa^i e_t^\beta \bar{h}_t - e_t^i w \bar{h}_t \geq w \bar{h}_t \quad (4.17)$$

More precisely, for an individual who decides to invest in education:

$$a^M \equiv (1 - p + p\eta)^{-1} \left(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}} \right)^{\beta-1} \quad (4.18)$$

where a^M is the ability threshold for an individual who is indifferent between investing or not. Taking the derivative of (4.9) with respect to p , we have

$$\frac{\partial a^M}{\partial p} = \left(\beta^{(\beta/(1-\beta))} - \beta^{(1/(1-\beta))} \right)^{(\beta-1)} (1 - p + p\eta)^{-2} (1 - \eta) < 0$$

An increase in p leads to a decrease in the threshold of potential emigrants. This is because an increase in p increases the expected lifetime income for a potential emigrant causing more individuals choose to become potential emigrants.

Similarly, for an individual to choose return migration, the following condition must hold:

$$[1 - p + p\mu\eta + p(1 - \mu)X] wa^i e_t^\beta \bar{h}_t - e_t^i w \bar{h}_t \geq w \bar{h}_t \quad (4.19)$$

For an individual who decides to return:

$$a^{RM} \equiv [1 - p + p\mu\eta + p(1 - \mu)X]^{-1} \left(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}} \right)^{\beta-1} \quad (4.20)$$

where a^{RM} is the ability threshold for an individual who is indifferent between returning or not. Further, his expected lifetime income as a potential returnee must be equal or higher than his expected lifetime income as a potential migrant. i.e.

$$[1 - p + p\mu\eta + p(1 - \mu)X] wa^i e_t^\beta \bar{h}_t - e_t^i w \bar{h}_t \geq (1 - p + p\eta) a (e_t^i)^\beta e_t^\beta \bar{h}_t - e_t^i w \bar{h}_t \quad (4.21)$$

or $X \geq \eta$ equivalently. X depends on an individual's ability to learn. When S is far be-

hind R on a development ladder, the industries in S that need upgrades require relatively lower skills. While S climbs up the ladder, the industries that need upgrades requires relatively higher skills. In terms of the model setup, S demands the individuals from lower to higher abilities while climbing up the ladder. When S demands higher abilities, $X = \chi_1 a^i$, where $\chi_1 \in \mathbb{R}^+$ denotes the payoff per unit of human capital per unit of ability. Therefore, only when his ability is high enough to offset the wage in R he would return to S. This confirms the western-eastern European migration pattern: many highly skilled migrants return to their motherlands Mayr and Peri (2008). When S demands lower abilities, $X = \frac{1}{\chi_2} a^i$, where $\chi_2 \in \mathbb{R}^+$ denotes the payoff per unit of human capital per unit of ability. Therefore, only when his ability is low he would return to S. This reflects the facts that it is the relatively low abilities who return to S (Borjas and Bratsberg, 1996). Hence, whether a returnee comes from the relatively high or low ability distribution depends upon the demand in S. Taking the derivative of (4.12) with respect to p , we obtain $\frac{\partial a^{RM}}{\partial p} = (-1) [1 - p + p\mu\eta + p(1 - \mu)X]^{-2} [-1 + \mu\eta + (1 - \mu)X] \left(\beta^{(\beta/(1-\beta))} - \beta^{(1/(1-\beta))} \right) < 0$. An increase in p leads to a decrease in the threshold of potential returnees. This is because an increase in p increases the expected lifetime income for a potential returnee and more individuals choose to become a potential returnee.

4.3 Balanced Growth Path

The aggregate human capital remaining in S at time t is:

$$H_t = \int_{\underline{a}}^{a^M} \bar{h}_t \Gamma(a^i) da + (1 - p) \int_{a^M}^{a^{RM}} h_t \Gamma(a^i) da + \int_{a^{RM}}^{\bar{a}} h_t \Gamma(a^i) da = \bar{h}_t \Gamma(a^i) \Omega \quad (4.22)$$

Where $\Gamma(a^i)$ denotes the population density at a^i and

$$\begin{aligned}\Omega &= (a^M - \underline{a}) + \frac{1-\beta}{2-\beta} (1-p)(1-p+p\eta)^{\frac{\beta}{1-\beta}} \beta^{\frac{\beta}{1-\beta}} \left[(a^{RM})^{\frac{2-\beta}{1-\beta}} - (a^M)^{\frac{2-\beta}{1-\beta}} \right] \\ &+ \frac{1-\beta}{2-\beta} [1-p+p\mu\eta + p(1-\mu)X]^{\frac{\beta}{1-\beta}} \beta^{\frac{\beta}{1-\beta}} \left[(\bar{a})^{\frac{2-\beta}{1-\beta}} - (a^{RM})^{\frac{2-\beta}{1-\beta}} \right]\end{aligned}\quad (4.23)$$

The remaining population in S is:

$$\begin{aligned}L_t &= \int_{\underline{a}}^{a^M} \Gamma(a^i) da + (1-p) \int_{a^M}^{a^{RM}} \Gamma(a^i) da + \int_{a^{RM}}^{\bar{a}} \Gamma(a^i) da \\ &= \Gamma(a^i) (-\underline{a} + pa^M - pa^{RM} + \bar{a})\end{aligned}\quad (4.24)$$

Hence the average human capital of teachers in t+1 is

$$\bar{h}_{t+1} = \frac{H_t}{L_t} = \frac{\bar{h}_t \Gamma(a^i) \Omega}{\Gamma(a^i) (-\underline{a} + pa^M - pa^{RM} + \bar{a})}\quad (4.25)$$

Assuming population grows at n , the aggregate human capital remaining in S at time t+1 is:

$$H_{t+1} = \frac{\bar{h}_t \Gamma(a^i) \Omega}{\Gamma(a^i) (-\underline{a} + pa^M - pa^{RM} + \bar{a})} \Gamma(a^i) (1+n) \Omega\quad (4.26)$$

Finally, the growth rate of the aggregate human capital remaining in S, and thus the growth rate of this economy is:

$$g_H = \frac{H_{t+1}}{H_t} = \frac{\bar{h}_t \Gamma(a^i) \Omega}{\Gamma(a^i) (-\underline{a} + pa^M - pa^{RM} + \bar{a})} \Gamma(a^i) (1+n) \Omega \quad (4.27)$$

$$= \frac{(1+n) \Omega}{-\underline{a} + pa^M - pa^{RM} + \bar{a}}\quad (4.28)$$

5 COMPUTATIONAL EXPERIMENTS

Chapters three and four provide two new theoretical models. In this chapter we, then, turn our attention to examining how good these two models are in fitting and predicting real world cases. However, as stated by (Luo and Wang, 2002) “The data available on HRST (human resources in science and technology) stocks and flows are extremely limited and inconsistent.”, the statistics about highly skilled migrants and returnees are the main concern. Because of this concern, we simply perform the computational experiments.

The ability to learn, ϕ in the first model and a in the second model, is inherent and the probability of migration, $P_{migration}$ in the first model and p in the second model, is given. Hence, both the ability to learn and the probability of migration are exogenous variables in these two models.

In the first model, the rate of return of financial assets is tied to the ability to learn on the balanced growth path. When the ability to learn decreases because of the out-migration of the skilled labor, the wage adjusts to ensure that the financial return matches the reduced ability to learn. Furthermore, both the time preference, ρ , and the coefficient of relative risk aversion, θ , are also exogenous variables. In the second model, the education productivity, β , the wage differential, η , the population growth rate, $1 + n$, the upper and the lower boundaries of the ability to learn distribution, \bar{a} and \underline{a} , the probability of migration, p , the fraction of working time in the receiving country, μ , and the payoff after returning under investment opportunities scenario, χ , are exogenous variables. So, in the calibration section, we can arbitrarily change the values of these parameters and see how the models response.

5.1 Experiments of the First Model

Following the standard procedure, we begin this section by calibrating parameters of the first model. Next, sensitivity tests are performed to build up the confidences of using this model. We, then, use the calibrated model to simulate the migration behavior.

5.1.1 Parameters Calibration

Key equations (3.79) and (3.90) in the model show the relationship between the representative individual's ability, the total migration effect $\frac{\dot{b}}{b} + \varphi$ and the economic growth rate γ . So, in this section, we focus on calibrating parameters used in these two equations. They are r , ρ , θ , φ , δ , μ , and γ .

The interest rate r is commonly used in the growth literature. It has value from 0.02 up to 0.14. The most commonly used value is 0.07 in Mehra and Prescott (1985) which represents the average rate of return on the stock market over the last century and we will use this value as a benchmark. The representative individual's ability, φ , is equal to the interest rate in this model. φ should have the value from 0.02 to 0.14 and the benchmark value is 0.07. ρ is 0.02 in closely related work like (Strulik, 2007) and we will use this value as benchmark. Following (Strulik, 2007), benchmark θ takes the value of 2.45.

From (3.79) and benchmarks above, the economic growth rate is about 0.0204. This economic growth rate is close to the observed one for the U.S. economy over the past 10 years. For the traditional Uzawa-Lucas human capital accumulation behavior, which is (3.22), these benchmarks indicate that the representative individual devotes 0.2857 of his time into human capital accumulation. This is roughly equal to the commonly accepted one-third of time. For the modified human capital accumulation behavior, which is (3.64), δ is one as other parameters being benchmarks. Recall that $h_t = [1/(\delta + 1)]\bar{h}_t$. The representative individual's human capital level is half of total human capital level available. These parameters are summarized as Table 1.

Because there is no uncertainty (i.e. no stochastic process) associated with the eco-

Table 1: Benchmark parameters of first model

Symbol	Definition	Baseline	Range	Source
δ	Adjusted learning effort factor	1		Calibration
ϕ	Learning ability = r	0.07	0.02-0.14	(Mehra and Prescott, 1985)
μ	Learning effort	0.2857		Calibration
r	Interest rate	0.07	0.02-0.14	(Mehra and Prescott, 1985)
ρ	Discount rate	0.02		(Strulik, 2007)
θ	Willingness to shift c b/t periods	2.45		(Strulik, 2007)
γ	Econ. growth rate	0.0204		Calibration

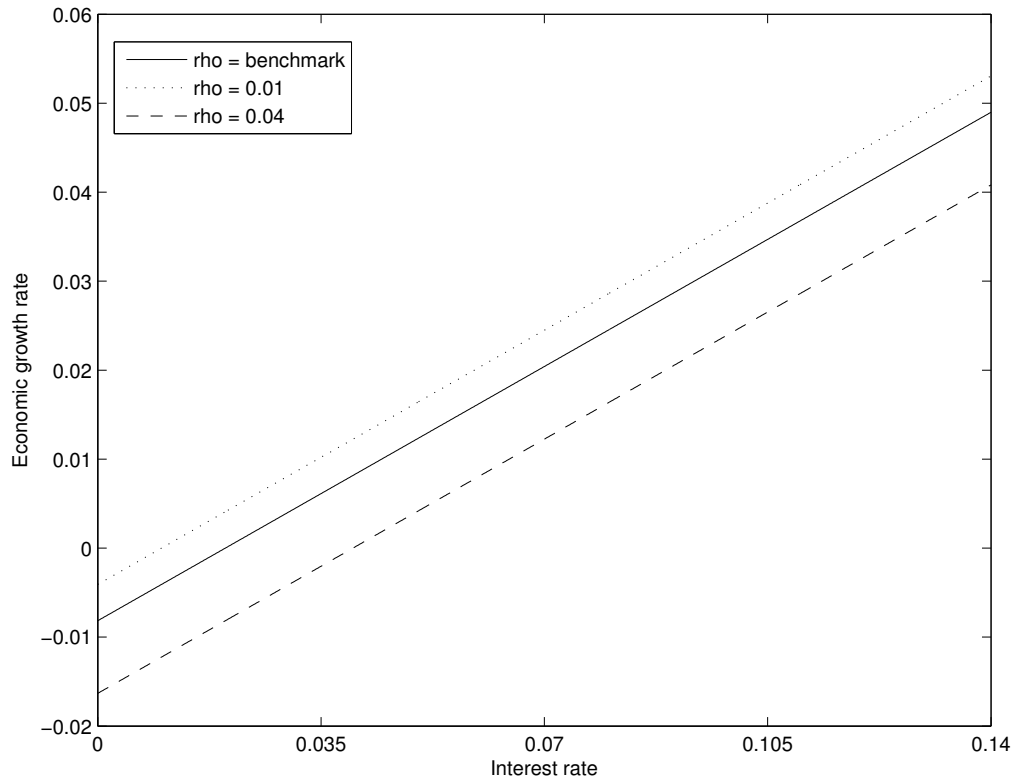
economic growth rate in this model, it is impossible for us to perform common technique in calibration - to generate values using the model and compare them with the observed values from data. Without uncertainty, we, then, turn into sensitivity tests.

5.1.2 Sensitivity Test

In this section, we perform three sensitivity tests. First of all, we test the sensitivity of the economic growth rate γ to a change in the interest rate r . In theory, a higher r means a higher ϕ . As ϕ increases, an individual acquires more human capital given the same μ . This means a higher human capital accumulation rate, a higher innovation rate, and a higher γ . Figure 5.1 simply depicts this idea. As r increases from 0.035, which is half of the benchmark value, to 0.14, which is twice of the benchmark, while holding θ at 2.45 and ρ at 0.02, γ increases from 0.0061 to 0.049. A significant change in the value of r can only lead to a small change in γ . Hence, γ is not sensitive to a change in r .

Second, we perform the sensitivity test of γ to a change in discount rate ρ . When ρ is higher, more final goods are been discounted. With the same amount of r and θ , higher ρ means lower γ . Figure 5.1 depicts this situation with θ being the benchmark value and r varying. When the value of ρ is 0.01, θ is 2.45, and r is 0.07, γ takes the value of 0.024. When the value of ρ is doubled at 0.04, θ is 2.45, and r is 0.07, γ takes the value of 0.012.

Figure 5.1: Relationship of economic growth rate and interest rate

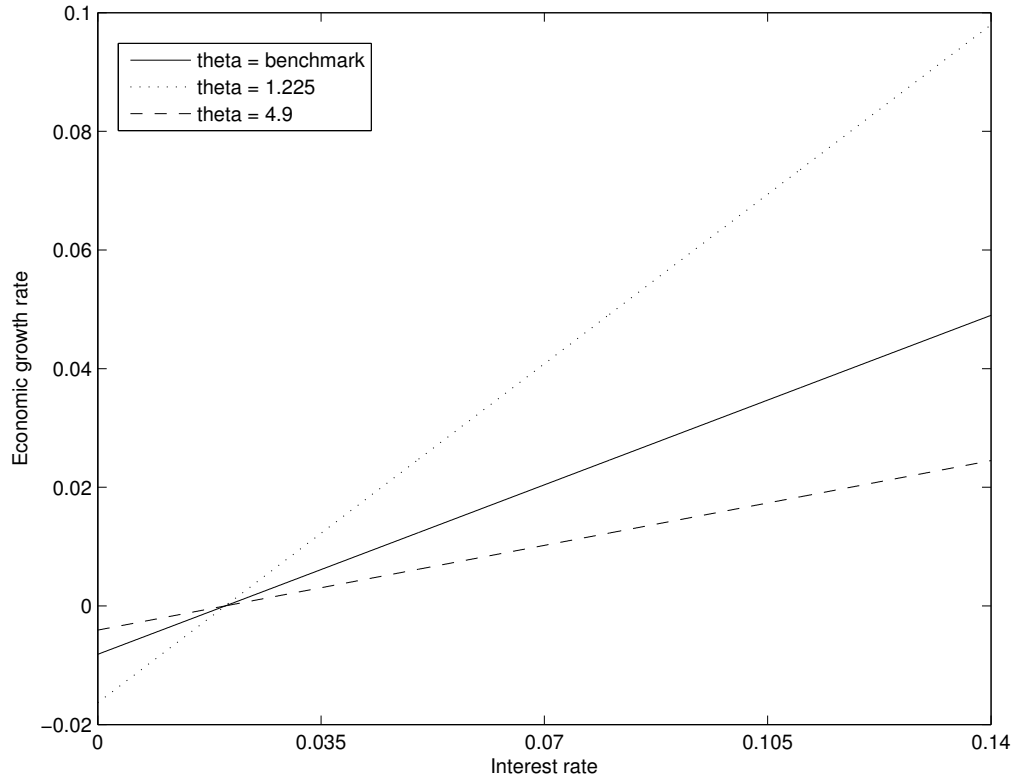


According to these values, γ is insensitive to changes in ρ . A significant change in the value of ρ will only lead to a relatively small change in the value of γ .

Third, a sensitivity test of θ is performed. Figure 5.2 depicts that an increase in the value of time preference θ increases the willingness to shift consumptions over time. This means c_t and, thus, γ at any point in time are smaller. A country with high θ is a country with more conservative view toward current consumption, for example - Japan. A half value of θ gives 0.041 of γ as r is 0.07 and ρ is 0.02. This is only 0.021 higher than the benchmark value. A doubled amount of θ gives 0.01 of γ as r is 0.07 and ρ is 0.02. This is only 0.01 lower than the benchmark value. Hence, we can say that γ is relatively insensitive to changes in θ . This is especially true when the value of r lies around 0.02.

Figure 5.3 provides another look of the insensitiveness of γ with respect to changes in θ . When r is 0.07 and ρ is 0.02, the curve is relatively flat for the value of θ between 2.45

Figure 5.2: Relationship of γ and r with changes in ϑ



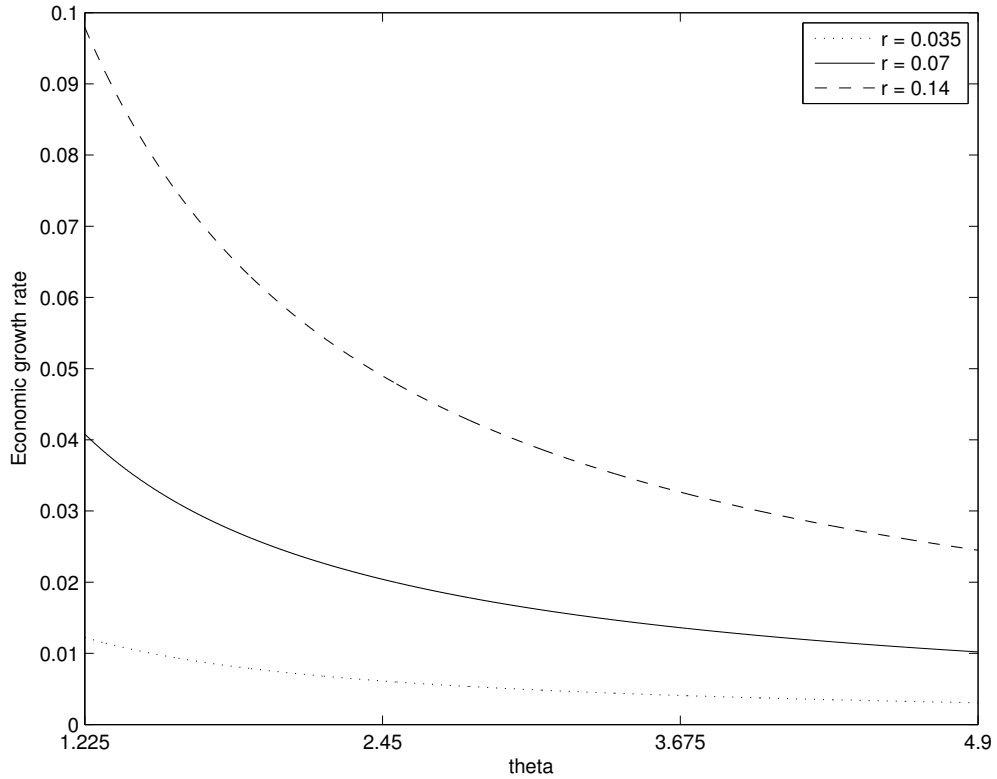
and 4.9.

From the sensitivity tests above, we know that the first model is insensitive around the benchmark values. This should give us some confidences to perform the simulation of emigration without worrying about the choices of benchmark values of parameters.

5.1.3 Model Simulation

In this section, we turn our attention to migration behavior simulation. Recall that the economic growth rate with migration is $\gamma = \frac{\dot{b}}{b} + \varphi - \rho$. With the expected wage rate $b_t w_{sending,t} = (1 - P_{migration})w_{sending,t} + P_{migration}w_{receiving,t}$, the instantaneous changes in expected wage rate is written as $\frac{\dot{b}}{b} = \frac{\dot{P}_{migration} (w_{receiving,t}/w_{sending,t})}{1 + P_{migration} (w_{receiving,t}/w_{sending,t} - 1)}$. Both a higher w and a higher probability of migration $P_{migration}$ could contribute to a higher expected wage rate and encourage the representative individual to acquire more human capital. The economy grows at the rate of

Figure 5.3: Relationship of γ and ϑ with changes in r



human capital growth. Hence, with a higher expected wage rate, the economy grows faster. We set values of w as 2, 4, 6 and 8, and set $P_{migration}$ from 0% all the way up to 20%.

We also consider several types of migration. The first type is that all migrants come from the high end of the ability distribution. A good example for this type of migration is Taiwan. Only those individuals who graduated from college and wanted to have further education had opportunities to cross the border in 60s, 70s, and 80s. The second type is that most of migrants come from higher end of the ability distribution and some of migrants come from rest of the ability distribution. One example of this type is South Korea. Many Koreans enter U.S. through student visa. After graduated from college, they switch their student visa to working visa. At the same time, some Koreans are low skilled labors. They own grocery stores or family restaurants in U.S. The third type is the one that migrants are drawn randomly across the distribution of ability to learn. The example of this type of

migration is Italian Americans. They are well represented in a wide variety of occupations and professions.

Obviously, the first type of migration does most damage for the representative individual's ability to learn in the sending country. All the migrants come from high ability distribution. The average ability for the entire S after migration is the lowest among these three types. On the other hand, the third type creates no damages on ϕ in S. Because migrants come from whole distribution and the average ability for S after migration is not affected by migration out-flow.

Figure 5.4 depicts the first type of migration with different w and $P_{migration}$. Clearly, as w increases, γ increases as well for any given $P_{migration}$. Also, for any given w , as $P_{migration}$ increases γ increases.

Figure 5.4: Relationship of γ and $P_{migration}$ with different wage ratios

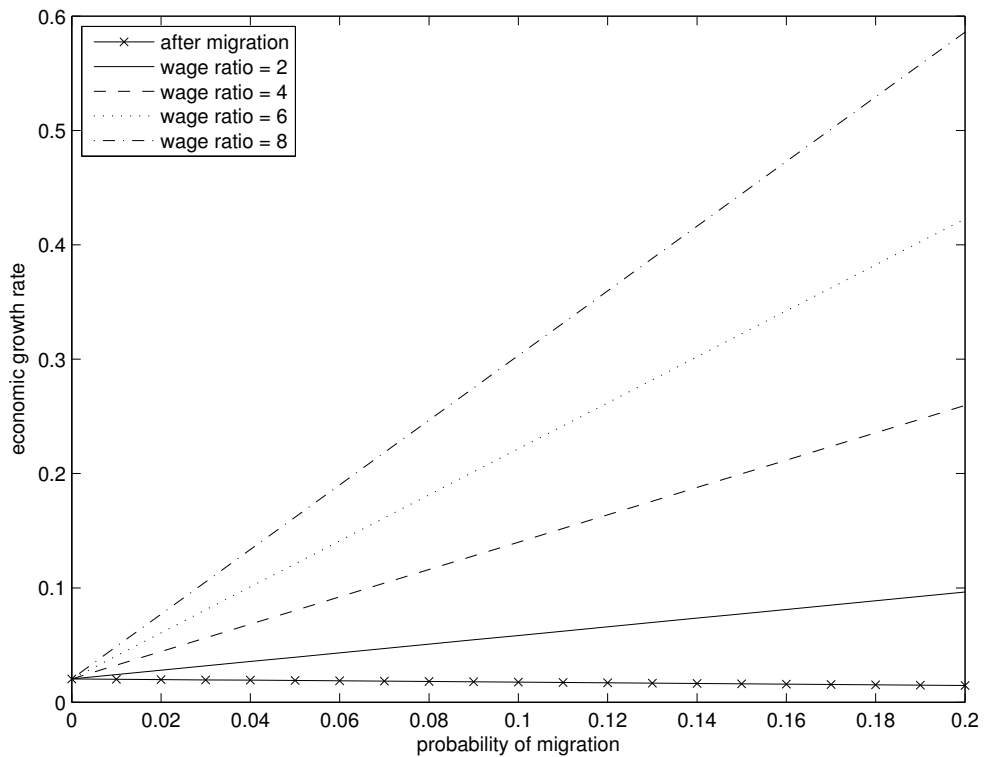
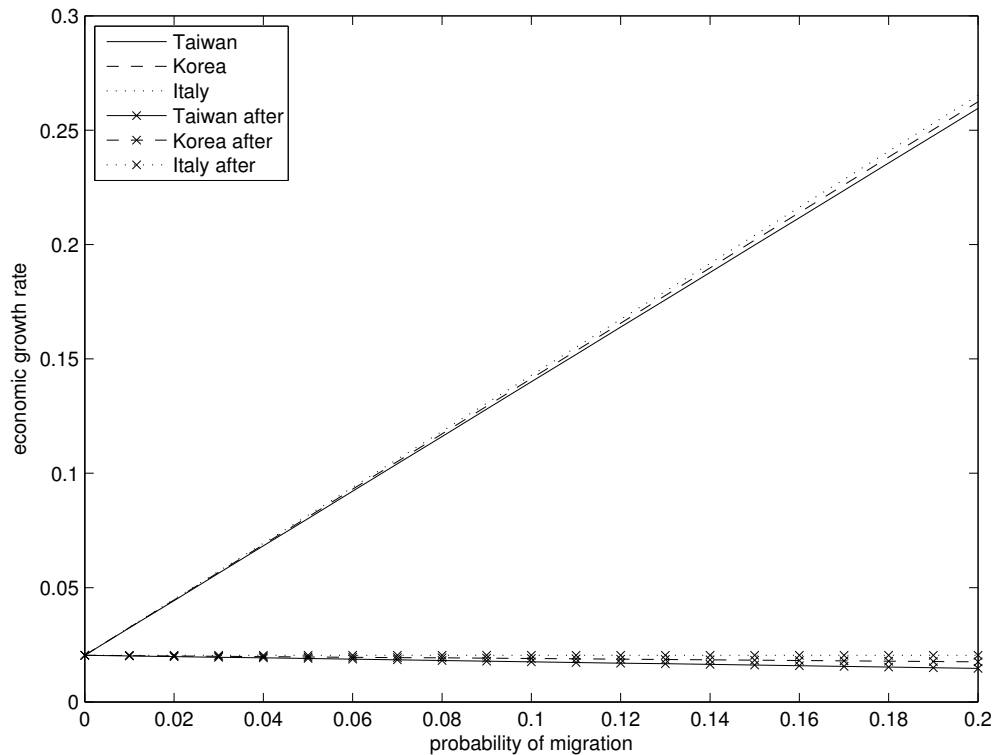


Figure 5.5 depicts three different types of migration with w equals to 4. An increase in

$P_{migration}$ can produce a higher γ for each type of migration. However, type one does have a lowest γ and type three does have the highest one across these three types of migration for any given $P_{migration}$. Hence, S should prevent from massive migration of high abilities. Although minor, it does damage γ most among these three.

Figure 5.5: Relationship of γ and $P_{migration}$ with three types of migration



However, one period after the initial changes in the $P_{migration}$, γ is always lower than the original rate for type one and type two without further changes in $P_{migration}$. This can be seen in both figures 5.4 and 5.5. Again, the first type of migration has the lowest γ after one period. One plausible explanation is that after the change happens, the representative individual instantly readjusts human capital accumulation rate to a lower level accordingly. Because when he faces the $P_{migration}$, he faces a higher expected human capital return and responds to this opportunity by increasing his human capital. After the initial changes in the $P_{migration}$, there is no further change in his expected human capital return. His human

capital accumulation rate returns to $\frac{\varphi-\rho}{\theta}$. However, because some individuals emigrate to R successfully, φ changes according to the type of migration. For type one and type two, φ is lower. This causes the human capital accumulation rate and φ to be lower than the initial one. For type three, φ remains unchanged. Hence, the human capital accumulation rate and γ remain unchanged.

In the model, we assume that each individual lives forever. In reality, an individual ages over time. If he starts to work at the age 18 and retires at 65, he contributes to economic growth for 47 years. In an economy containing individuals with different ages and without population growth, each individual would only increase his human capital accumulation rate once. After that, he would reduce his human capital accumulation rate for the rest of his life. In other words, only those individuals with certain age(s) would have a higher human capital growth rate. For the rest of working individuals, they would likely have a lower human capital growth rate. When we calculate γ across ages, we have to include this age issue. After accounting for ages, the wage difference must be sufficiently large to have a beneficial brain drain for S. For example, with benchmarks and type one migration, the wage difference must be larger than 4!

5.1.4 Policy Implications

Results from our simulation section give us some practical policy advices. First, a higher $P_{migration}$ could lead to a higher γ . S should try to raise $P_{migration}$ by supporting out-migration although in many cases $P_{migration}$ is controlled by R. Second, a larger w could lead to a higher γ . If S is relatively poor, it might expect to have a higher γ when opening to emigration. This suggests that the government of S should permit migration at early stage of economic development especially before the economy takes off. Third, the types of migration do matter. S should try to prevent highly intelligent individuals from migration. A more ability-diverse migration flow has less impact over the economy. Forth, after adjusting for $P_{migration}$, γ is lower than the initial one. With the consideration of ages, S should

try to encourage potential migrants emigrating to a more advanced country with higher w .

5.2 Experiments of the Second Model

The theoretical results in chapter 4 suggest that an increased probability of migration has two opposite effects on the overall human capital of the sending country. First, it creates the economic incentive to emigration - a higher expected income for potential emigrants and potential returnees. So, more individuals are willing to become potential permanent migrants and potential returnees. Second, it creates the incentive for potential emigrants and potential returnees to accumulate more human capital - a higher wage per human capital for both potential emigrants and potential returnees. Hence, potential emigrants and potential returnees voluntarily invest more into education. A lower threshold for potential emigrants, a^M , results in a lower overall human capital stock remaining in the sending country. And a lower threshold for potential returnees, a^{RM} , a higher education investment for potential emigrants, e^M , or a higher education investment for potential returnees, e^{RM} , results in a higher overall human capital stock. This raises the question of which effect dominates and how large the effects are quantitatively. Therefore, this section is designed to answer these questions.

5.2.1 Parameters Calibration

Burgess and Haksar (2005) suggests that international migration has been a notorious characteristic of the Philippine's economy. Here, we choose parameters of the model such that the balanced growth path resembles empirical features of the Philippine's economy. Following Marchiori et al. (2009), O.E.C.D. countries are chosen to represent the receiving countries.

We calibrate the model under the assumption that one period has the length of 30 years. The parameter β is 0.2 in Chen (2009) and Card and Krueger (1992) and 0.198 in Johnson and Stafford (1973). Since both these two values are practically the same, we set it equal

to 0.2. The wage differential between S and R, η , is set equal to 5.6654 as previously used in Marchiori et al. (2009). Because the population growth rate during 1995 and 2000 was 3.21% annually, $(1+n)$ is equal to 2.58. The average ability is set at 1.98 to match the annual economic growth rate 3.21% or 1.54 over 30 year-period. This gives the highest ability to learn, \bar{a} , a value of $1.98 \times 1.5 = 2.97$ and the lowest, \underline{a} , a value of $1.98 \times 0.5 = 0.99$. The probability of migration p is 8.6% in Marchiori et al. (2009) and 8.2% in Chen. We arbitrarily set the emigration rate equal to 8.2% in the baseline case. Also in the baseline case, the fraction of working time spending in R, μ , is 0.5. This fraction represents a returnee spending 15 years in S and another 15 years in R. The payoff after returning to S, X , takes the value of χa^i . i.e., the expected payoff for an emigrant with ability a^i after returning depends upon his ability and a shifting factor $\chi > 0$. This captures the idea that the higher ability of an individual, the higher chance he could succeed in business.

5.2.2 Results

In this subsection, we perform several experiments and report the results in Table 2.

The top-left ($\mu = 0.5$), top-right ($\mu = 0.25$), and lower-left panel ($\mu = 0.75$) show impacts of increases in p from 8.2% to 16.4% and to 32.6% on a^M , a^{RM} , e^M , e^{RM} , and g_H respectively. e^M and e^{RM} are reported without the multiplication of ability. In so doing, for a potential emigrant or returnee, his education efforts vary without consideration of his ability. i.e. We only care about whether he puts more efforts into education or not. The lower-right panel shows impacts of increases in χ . g_{H1} represents the human capital growth rate and economic growth rate without return-migration. The annual growth rates of aggregate human capital are reported in parentheses.

First, we analyze the impacts of increases in the probability of migration p from 8.2% to 16.4% and to 24.6% respectively. Clearly, responding to the changes in p , both the thresholds for a potential emigrant or returnee are lowered. Because the expected income for a potential emigrant and returnee increase, more individuals can afford to become a

Table 2: Impacts of increases in p and X

p	8.20%	16.40%	32.60%
d^M	1.193	0.9344	0.768
d^{RM}	2.7923	2.7664	2.7498
e^M	0.2005	0.2721	0.3477
$(e^{RM})^{1.25}$	$0.2301 + 0.0163a^2$	$0.2601 + 0.0336a^2$	$0.2902 + 0.0507a^2$
g_H	3.2624 (4.02%)	3.4109 (4.17%)	3.5738 (4.34%)
g_{H1}	3.2513 (4.01%)	3.3825 (4.15%)	3.5205 (4.28%)

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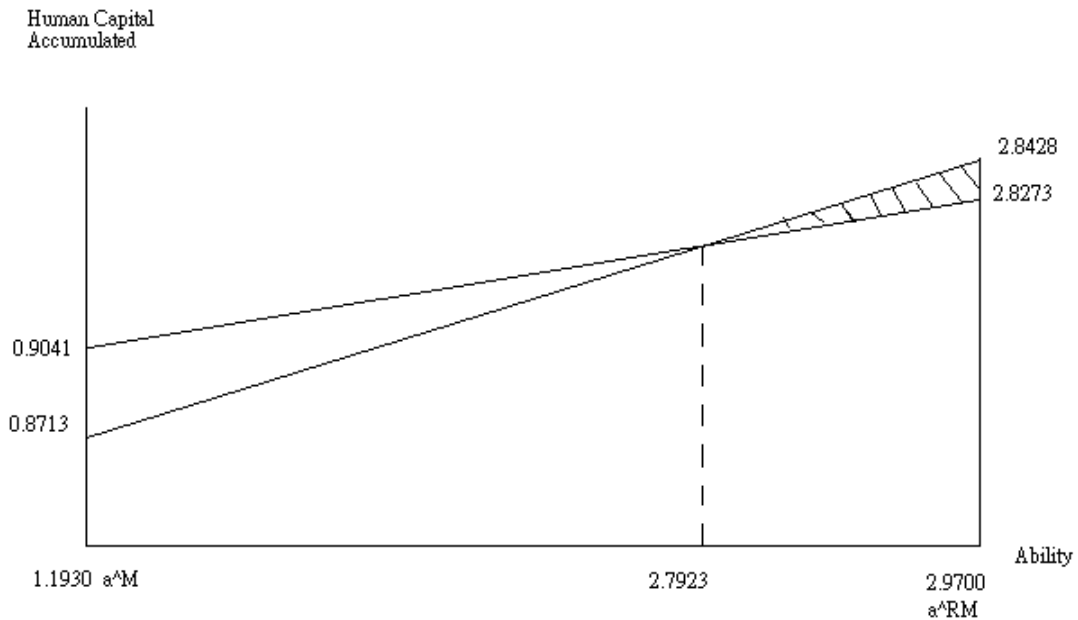
potential emigrant or returnee. Because the expected wage rate per human capital increases, the return to the education investment becomes higher. Each potential emigrant or returnee is willing to invest more into education and to accumulate more human capital.

The top-left panel of Table 2 shows that the human capital growth rate with and without return-migration, g_H and g_{H1} , increase when the probability of migration increases. This finding is similar to previous brain drain literature such as (Beine et al., 2001), (Mayr and Peri, 2008), Chen (2009), and Marchiori et al. (2009). The combined impact of a decrease in aggregate human capital resulting from a lowered a^M and an increase in aggregate human capital resulting from lowered a^{RM} , e^M , and e^{RM} is an increase in the aggregate human capital, and thus the aggregate human capital growth rate g_H . Hence, with plausible parameter choices, skilled labor emigration could generate brain gain as previous literature suggested. Notice that g_H is always higher than g_{H1} . With return-migration, returnees join the labor market in the sending country and contribute to the overall human capital stock.

Second, we change the fraction of time spends in R, μ . Comparison across top-left, top-right, and lower-left panels of Table 2 shows that the threshold and education effort for a potential emigrant won't be affected by changes in μ . However, the threshold for a potential returnee decreases and the education effort increases when μ increases from 0.25 to 0.75. When facing the same payoff after returning, a shorter stay in R allows a returnee to have a higher expected lifetime income. More individuals find themselves earning more with return-migration decision and each potential returnee is willing to invest more in education.

Next, we shift the payoff after returning, χ , from 2.0289 to 2.1668 and to 2.3248, equivalently, 10%, 20%, and 30% of emigrants return to the sending country, respectively. The lower-right panel of Table 2 documents the results. When χ increases, a potential returnee is willing to invest more into education and more potential emigrants become potential returnees. The consequence of this change is that the overall human capital accumulates at a higher rate and the economy grows faster.

Figure 5.6: Difference between wage premium and investment opportunity



Finally, we compare the difference between wage premium and investment opportunities in Figure 5.6. With baseline parameters, when the sending country, S, offers a full wage subsidy, which is the maximum amount of the wage premium, in attracting the top 10% of emigrants back to S, there is no incentive for a potential returnee to accumulate more human capital. However, the investment opportunity could create extra incentives for the potential returnee to accumulate more human capital. The shaded area represents the overall human capital gain with investment opportunity.

6 CONCLUSION

This dissertation contains two models. The first model investigates the possibility of a beneficial brain drain arising from the skilled labor out-migration, and the second model investigates the possibility of reverse brain drain arising from the improvement of economic environment.

6.1 The First Model

The first model integrates R&D and human capital accumulation with a modified human capital accumulation behavior that specifies the source of knowledge – previous innovations. An individual learns from previous innovation and creates new human capital. Then, he utilizes this new human capital in R&D activities and discovers new innovation. The representative individual's ability to learn, rather than the exogenous education productivity, determines the growth rates of human capital accumulation, innovations, and, thus, economic growth. There is no scale effect. When considering the skilled labor out-migration, the calibration results show that an increased probability of migration or wage differential between two countries could improve the sending country's economic growth rate, the more migrants that come from the high end of the ability distribution the more damaging the impact on the sending country, and potential migrants accumulate human capital faster before migration and slower afterwards.

The first model can be extended in many directions. First, equation (3.1) can be rewritten as $Y_i = F_i^{1-\alpha} \left[\sum_{j=1}^N (q^{kj} x_{ij})^\sigma \right]^{\frac{\alpha}{\sigma}}$. This equation is more general. Equation (3.1) is a

special case of this equation when $\alpha = \sigma$, although results are similar. Second, this model contains only one state variable. The inclusion of another state variable allows us to study the dynamics in the short run as well as long run. Third, in the model, we make the assumption that each individual lives forever. This assumption simplifies the mathematical derivations of the model. But it also creates the drawback that when facing an increase in the probability of migration the representative individual responds to the change only once. The economic growth rate increases once and stays low forever. An overlapping-generation assumption should allow the representative individuals from each generation to respond to the probability of migration once. This should give persistent increases in the human capital accumulation and economic growth rate. Forth, the assumption that the ability distribution is uniform is over simplified. Instead, we can assume a normal distribution. However, these modifications could lead to mathematical complications. Fifth, another direction relies on the data availability. In this dissertation, we test the model by using computation due to the limitation of the data. In the future, when the data becomes available, we can use econometrics technique to test the model.

6.2 The Second Model

Most of the theoretical literature on brain drain and growth has concentrated on the skilled labor emigration through the channel that uncertainty over migration motivates individuals to invest more in education. This dissertation investigates another plausible channel of brain gain from skilled labor's return-migration. The sending country demands new technologies and managerial skills to upgrade industries while climbing up the development ladder. Often, investment opportunities emerge. Skilled emigrants are likely to have acquired technologies or managerial skills in the receiving country, hence, better chances to succeed than stayers. They return to the sending country and contribute to the sending country's overall human capital.

The second model we present is a simple overlapping generation endogenous human

capital growth model with self-selection migration decisions. Our theoretical results show that an increase in the probability of emigration create two opposite impacts on the formation of human capital. First, more individuals emigrate to the receiving country. This is the negative impact. Second, more individuals become potential returnees, and a potential emigrant or returnee invests more in education. All three of these are positive impacts. Our calibration results show that an increase in the probability of emigration results in an overall human capital gain. Further, emerging from the economic development progress, investment opportunities serve as vehicles of brain gain. Although, both wage premium and investment opportunities could serve as vehicles in attracting emigrants back to the sending country, investment opportunities could induce additional human capital accumulation while wage premium couldn't. The duration of staying in the receiving country is also an important determinant of the human capital formation. A shorter stay in the receiving country means higher expected income for potential returnees which induces more emigrants to return to the sending country and contributes more to the human capital formation. Such a feature enables the model to reproduce the evolutionary fact that emigrants stay in the receiving country shorter and shorter during the sending country's development period.

There are several directions to extend this model. First, we could relax the assumption of the smallness of the sending country. The capital return and the wage rate per human capital are flexible. Hence, we can discuss about effects of the sending country in catching up with the receiving country on the human capital formation and the return-migration flow. Second, we could model technology spillovers stemming from scientific parks into payoffs after returning. Therefore, we could have a more complete model and obtain the threshold of massive return-migration. Third, the uniform ability distribution could be replaced by a normal distribution. This can bring the model closer to the reality. Forth, we could relax the assumption of a fixed duration of staying in the receiving country. Individuals could choose different amounts of education and different durations of staying.

6.3 Comparisons of the First and the Second Models

Both the first model and the second model can be classified as endogenous growth models. The first model is an integrated R&D and human capital endogenous growth model and the second model is simply an endogenous human capital growth model. Both of them focus on the issue of skilled labor out-migration and the economic growth from the sending country's perspective. Both of them predict a relatively higher economic growth rate for the sending country when the wage differential between the sending country and the receiving country is larger. However, there is the technology progress in the first model but not in the second model. Further, the interest rate varies in the first model. In the second model, the sending country is assumed to be a small country and its interest rate is fixed at world's interest rate level.

The purposes of these two models are different. The main purpose of the first model is to show that, in the context of R&D growth modelling, skilled labor out-migration could still contribute to the sending country's economic growth even without the scale effect. The main purpose of the second model is to show that investment opportunities and wage subsidies have different effects over the human capital formation in the sending country.

The model structures are also different. The first model is built upon a typical endogenous R&D growth model - Barro and Sala-i-Martin's model. Then, we follow Arnold (1998) to incorporate the human capital growth. Finally, we modify the human capital accumulation behavior to make it truly endogenous and scale effect free. And in building the second model, we follow the tradition in the brain drain literature and assume that each individual can choose where to work and how much human capital to acquire (i.e. self-selection) - a feature that is missing in the first model (where an representative individual decides the amount of human capital to acquire). We also follow the tradition in adopting an overlapping-generation framework in the second model instead of forever-living framework in the first model.

Apart from these assumptions shared with other recent brain drain works, we allow each

individual to choose his own human capital level instead of his children's human capital levels in the second model. We also permits a successful emigrant to choose return-migration. Only a few works in the literature had touched this issue of reverse brain drain. The most unique feature not seeing in others lies on the mechanism that attracts emigrants' return to the sending country. Other brain drain works with return-migration combine investment opportunities and wage subsidies together as an incentive that lures the return-migration. In the second model, we distinguish between the effects of investment opportunities and that of wage subsidies over human capital formation in the sending country.

6.4 Policy Implications

Decades ago, Taiwan began to open for the emigration of those with high abilities. Aiming for most advanced countries such as Japan, Germany, England, and U.S, those Taiwanese college students studied harder than ever to compete with each other for government permits to study abroad after graduation. When Taiwanese government increased the number of permits, more and more Taiwanese attempted to go to colleges and then to study abroad. Most of the students did not return to Taiwan after receiving their degrees abroad. They found jobs and lived overseas. In spite of the fact that a large portion of these students succeeded in emigration, those who failed had also invested more into education and accumulated more human capital than otherwise. Many of these stayers became leaders who later on contributed to Taiwan in politics and economics significantly.

Mainly because of emigration policies, most of Taiwanese emigrants were skilled labor while Korean emigrants consisted of some non-skilled labor and some skilled labor. According to the Department of Homeland Security (D.H.S.) statistics, Korean immigrants made up the sixth-largest immigrant group and Korean students were the second most, only behind the Japanese, in the U.S. In 2003. 51.3% of Korean immigrants whose ages were 25 and up had a bachelor's degree or higher. On the other hand, 9.5% of Korean immigrants had no high school diploma and 20.7% had only high school diploma. It seems that

the composition of emigrants in Taiwan might have induced less additional human capital compared to S. Korea. After returning, they would have made more contributions and promoted a higher economic growth rate. Therefore, the strategy that sending out skilled labor and improving investment environment in attracting returnees carried out nicely for Taiwan.

As the wage differences between Taiwan and advanced countries became smaller, fewer Taiwanese students studied abroad. According to Bureau of International Cultural and Educational Relations R.O.C, only 12000 to 14000 students went to U.S. every year from 1990 to 2007. At the same time, the Korean society sees the experiences of studying abroad as valuable asset in their job searches. By living in an English-speaking country and by studying new technologies or managerial skills in an advanced country such as the U.S, many south Korean parents believe that their children will find higher-paying jobs either abroad or at home. According to D.H.S, there were 45413 Korean students in 1997 and 135265 in 2006 in the U.S. The difference in out-migration flows between Taiwan and S. Korea in the 1990s and 2000s could be one of the reasons that S. Korea grew 2.4 times while Taiwan grew 2.1 times higher since 1990 (in terms of P.P.P. per capita, C.I.A. Factbook).

Taking a closer look from the Taiwanese experience, we know that potential emigrants are most likely to target those countries with higher wage rate per human capital (equations (3.89) and (4.27)), potential emigrants accumulate more human capital (equation (3.90)) and more people are willing to be potential emigrants when facing an increased probability of migration (equations (4.9) and (4.12)), and brain drain is not necessarily detrimental to the sending country's economic growth (equations (3.90) and (4.27)). However, the direct evidence that brain drain led to a relatively higher aggregated human capital accumulation rate is still not evident. Further, the number of emigrants were much smaller compared to the entire population. Even if brain drain could lead to a relatively higher human capital accumulation rate for those potential emigrants, the effect of brain drain on the aggregative

human capital accumulation rate may not fully account for the increments of the economic growth rate between 1960 and 2000 in Taiwan.

High education cost for many people in the under-developed countries is the main concern that prevents them from acquiring more education. In the 1960s, education funding was only about 2.5% of Taiwanese GDP. Since then Taiwanese government started to subsidize heavily on education. In 1968, six-year compulsory education was extended to nine-year. The education funding increased to more than 4% of Taiwanese GDP in the 70s and 6% in the 80s. Taiwanese education attendments had improved significantly. Both human capital level and accumulation rate increased tremendously. The governmental education subsidy had a positive effect on the human capital formation. As previously mentioned, whether brain drain had an overall positive effect on the human capital formation for Taiwan is still unclear. However, the number of emigrants was small compared to the entire population of Taiwan. Brain drain had led to a lost of some talents while the governmental education subsidy had improved the majority of Taiwanese human capital accumulation. The negative effect on the human capital formation from losing talents was small compared to the positive effect from the governmental education subsidy. The positive effect dominated the negative effect. Consequently, Taiwanese human capital accumulation and economic growth rates were relatively higher than otherwise.

For the sending country, the main contribution of reverse brain drain is that skilled returnees bring back the knowledge acquired in the receiving country that the sending country needs and thus expand the knowledge base of the sending country. There are two ways to expand the knowledge base - enterprise innovations and the basic research. These two can also serve as economic incentives in reversing the brain drain effectively.

For Taiwanese emigrants, the sending country had relatively lower quality of intermediate goods compared to the receiving country. Some skilled emigrants could access to the key knowledge of the reproductions of intermediate goods at higher qualities. When a skilled emigrant brought back the key knowledge of the reproduction of an intermediate

good at a certain quality, this skilled emigrant could have higher probability to succeed in reproducing this intermediate good at that quality, i.e. he could have higher probability to improve the quality of this intermediate good to that certain quality. He could enjoy a higher expected monopoly profit over his counterparts, i.e. the expected monopoly profit for this skilled emigrant could be higher than or equal to his income in the receiving country. This skilled emigrant is willing to bring the key knowledge back to the sending country and to contribute to the sending country's knowledge base. To improve the probability and increase the expected payoff for the returnees, the government could create an environment such as scientific parks.

Similarly, some skilled emigrants could access to the key knowledge of the reproductions of scientific findings. When a skilled emigrant brought back the key knowledge of a scientific finding, this skilled emigrant could have higher probability to succeed in reproducing this finding. He could enjoy higher expected payoff from research fundings over his counterparts. When his expected payoff is higher than or equal to the income he would receive in the receiving country, he is willing to return to the sending country and to contribute to the sending country's knowledge base. The government could increase the basic research fundings to attract those scientists to return to the sending country.

In the 1980s, Taiwanese economy had reached a certain point that required technologies and managerial skills from advanced countries to upgrade industries and to sustain the high economic growth rate. Taiwanese government built scientific parks to enhance technology spillovers, by providing financial supports, and reducing taxes. All of these changes were designed to increase the expected payoff for the returnees. Emigrants began to return to Taiwan and contributed to the overall human capital stock. A good example is the Taiwan Semiconductor Manufacturing Company. The founder of this company, Dr. Morris Chang, brought the once top of the line knowledge of semiconductor back to Taiwan from Texas Instruments in 1987.

After Dr. Morris Chang and many others returned to Taiwan, the demand for engi-

neers increased significantly. This had direct impacts on the expected payoffs of returnees. Therefore, when more and more emigrants returned to Taiwan, potential returnees accumulated more human capital and more potential emigrants became potential returnees because of the higher expected lifetime income after returning (equations (4.15) and (4.16)). Furthermore, returnees spent less time staying in receiving countries. Shortly after acquiring technologies or managerial skills, they returned to Taiwan.

Meanwhile, Taiwanese government increased the amount of funding and salaries for those basic researchers. For example, the amount of education devoted to higher education as percentages of annually total education funding increased from 13% in the 1970s, to 18% in the 1980s, to 22% in the 1990s, and to 35% in the 2000s. Hundreds of basic researchers abroad returned to Taiwan and conducted research either in academic world or in independent research institutions. For example, Dr. Yuan-Tseh Lee, a Nobelist, had returned to Taiwan and became the president of Academia Sinica in 1994. These returned scientists had improved Taiwanese basic research capability by a great measure.

Resently, there is another wave of brain drain in Taiwan. Seeking for better local connections and investment opportunities in the Chinese emerging economy, many Taiwanese studies in China. With this new wave of brain drain, Taiwan could grow even slower compared to S. Korea in the future. And as more statistics become available, these theoretical models are expected to serve better predictors of the reality.

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