STATISTICAL EVALUATION OF WEATHER MODIFICATION: TARGET TWO-SAMPLE RUN METHOD

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Radmilo D. Markovic

Engineering Research Center
Colorado State University
Fort Collins, Colorado

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The distribution-free run method, devised for solving non-parametric problems of testing whether two continuous distributions are identical, was employed for statistical evaluation of weather modification at the river flow control level. The practical application of this two-sample method to the sequences of runs of ordered nonseeded and seeded annual river flows drained from a target basin indicated that the method is sensitive to differences both in shape and in mean between two distributions.

INTRODUCTION

In this paper, the term "weather modification" covers all activities concerned with the production of precipitation and resultant runoff. Thus this term includes artificial cloud modification brought about to induce rain from otherwise nonprecipitating clouds and/or to increase natural precipitation from such clouds by improving their precipitation efficiency. In the past, cloud modification has been performed almost entirely by introducing condensation or ice nuclei into cloud systems. Therefore, for all practical purposes, "cloud seeding," also means weather modification.

The statistical evaluation of weather modification must be based upon cautious mathematical and statistical analysis of data from many weather modification experiments. That is, the problem is determining whether these activities actually produced an increase in precipitation or runoff identifiable over a well defined target area.

Most statistical techniques used in the evaluation of weather modification attainments rely on the assumption that the variable describing the hydrologic phenomenon under consideration (precipitation and/or river flow) is normally distributed. This assumption is not so restrictive as it may seem at first sight, for three reasons. First, it has already been shown that, in many practical cases, the distribution of precipitation or of a river flow variable can best be approximated by normal distribution [Markovic, 1965]. Second, a transformation can often be found which will bring the observations close to the normal form and permit the application of the normal theory. Third, frequently means can be dealt with and then, in accordance with the central-limit theorem, the distribution of the sample means approaches normality as sample size increases if a population has a finite variance.

How important the assumption of normality in statistical evaluations of weather modification is depends on the magnitude and nature of any departure from normality and on the kind of statistical techniques being used. Highly skewed distributions, for example, can badly upset the level of significance of a one-tailed test even though they have only a small effect on a two-tailed test. Since an increase in precipitation and runoff due to weather modification experiments is anticipated a priori, the upper or one-tailed test is usually indicated. However, deviation from normality in the extreme tails is rather unimportant in significance testing. In general, deviations from normality cause fewer gross errors than do two other departures from the usual assumptions: lack of constancy of variance and lack of independence of observations [Brownlee, 1960].

Unfortunately, often it is not known whether a hydrologic variable's basic distribution is the kind to which the central limit theorem applies. Moreover, sometimes the approximation to normal distribution may not be good enough; then the resulting confidence intervals and the tests of hypotheses based on normal theory are not as accurate as supposed.

For cases in which conventional methods--based on the assumption of a normal distribution--are not applicable, an alternative method must be found. In recent years, techniques have been developed which assume only that the form of an underlying distribution is continuous and assume nothing about the form of that distribution. These techniques are known as distribution free methods. The observations of hydrologic variables certainly do have distributions with parameters: what one is free of is assumptions about the forms of distribution.

In this paper, the method used ignores the functional forms of parameters and of distribution functions for the basic variables. It can be applied to very wide families of distributions as well as to families specified by particular functional forms. The properties of this method, therefore, allow application of it to a large variety of man's activities in the field of weather modification as well as to the various control levels from which statistical evaluation of weather modification can be considered. These control levels include the levels of cloud phenomena, precipitation, and river flow control, corresponding to three particular stages in the general hydrologic cycle.

Since it has already been demonstrated [Markovic, 1967] that, under general conditions, river flow control level seems likely to evolve into the most accurate, most reliable, and most detectable way of control of the three levels, the information in the rest of this paper pertains exclusively to the river flow control level.

The fundamental variable at the river flow control level is the flow rate or flow discharge. The annual river flow averaged over a water year which starts usually on October 1 and lasts to September 30 of the next calendar year is the only variable that needs to be considered for this method of statistical evaluation of weather modification attainments. Meteorologically and hydrologically, annual river flow is very convenient because its time unit generally coincides with the complete hydrologic annual cycle of all the physical processes of seasonal nature.

CONSTRUCTION OF THE TEST

First, Q_i ($i = 1, 2, 3, \ldots, n$) denotes the ith sequential observation in a sample of n observations of annual river flows at a river gaging station registering the runoff drained from the target basin in the nonseeded period. Likewise, $Q_{(i)}$ ($i = 1, 2, 3, \ldots, n$) denotes the ith ordered observation in the same sample (i. e., the symbol $Q_{(1)}$ refers to the smallest of the n observations, $Q_{(2)}$ to the second smallest of the observations, and so on, while $Q_{(n)}$ refers to the largest). Representations of sequential and ordered observations, hydrograph and duration curve respectively, are graphed in Figure 1.

Similarly, Q^*_j ($j = 1, 2, 3, \ldots, m$) denotes the jth sequential observation in a sample of m observations of annual river flows from the same target basin but in the seeded period. Also, $Q^*_{(j)}$ ($j = 1, 2, 3, \ldots, m$) denotes the jth ordered observation in the target seeded

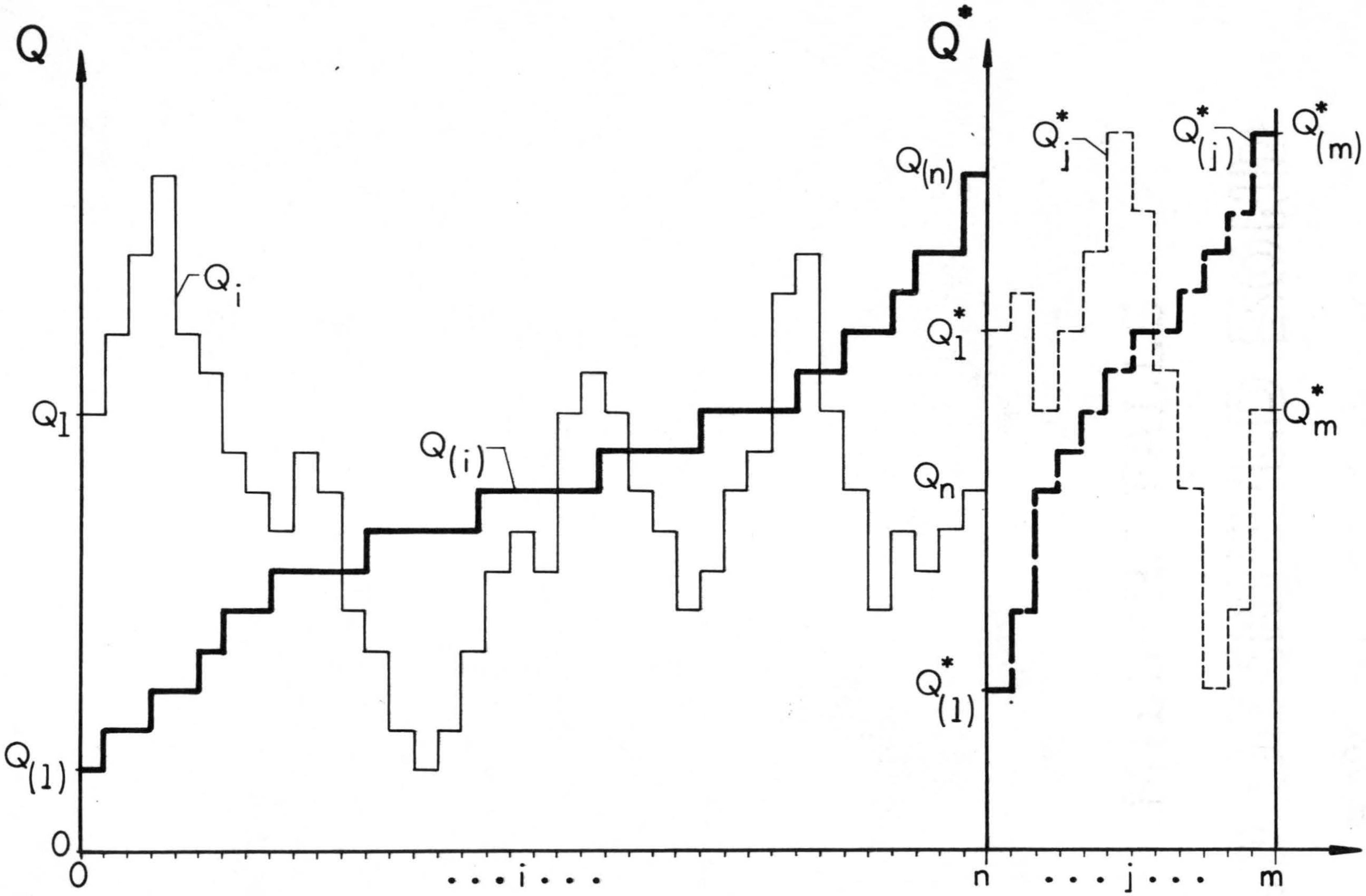

Fig. 1 Hydrographs of nonseeded and seeded river flows, Q_i and Q^*_j, with their corresponding duration curves, $Q_{(i)}$ and $Q^*_{(j)}$.

sample of annual river flows (Figure 1).

Next, the above two target samples, nonseeded [$Q_{(1)}$, $Q_{(2)}$, $Q_{(3)}$, ..., $Q_{(n)}$] and seeded [$Q^*_{(1)}$, $Q^*_{(2)}$, $Q^*_{(3)}$, ..., $Q^*_{(m)}$] are combined and arranged in order of magnitude (Figure 1). In this way, it is possible to obtain the following arrangement of ordered nonseeded and seeded annual river flows in the target basin:

$$Q_{(1)},\ Q_{(2)},\ Q^*_{(1)},\ Q^*_{(2)},\ Q_{(3)},\ Q^*_{(3)},\ Q_{(4)},\ Q^*_{(4)},\ Q_{(5)},\ \ldots \qquad (1)$$

The expression (1) starts with an arrangement of two Q's called the run of two Q's; then follows the run of two Q* 's, the run of one Q, etc. Altogether, seven runs are exhibited in (1). As can be observed, a run is a sequence of ordered observations from the same sample (such as nonseeded) bounded by observations from the other sample (such as seeded). For example, a run of Q's is a set of successive Q's closed off at both ends by Q*'s (except at the beginning and end of the sequence), and vice versa for a run of Q*'s.

It is obvious that, if two samples are from the same population (that is, if the seeding experiments had no effect whatsoever), the nonseeded and seeded observations will ordinarily be well mixed and the number of runs, η, will ordinarily be large (Figure 2 a). If the seeded experiments had a very strong effect upon annual river flows, the two samples would be taken from two distinct populations. If these two populations are widely separated so that their ranges do not overlap, the number of runs would be only two, $\eta = 2$ (Figure 2 b). In general, the larger the difference between the two populations, the smaller the number of runs. In other words, the difference between two populations tends to reduce the number of runs. Even if the two

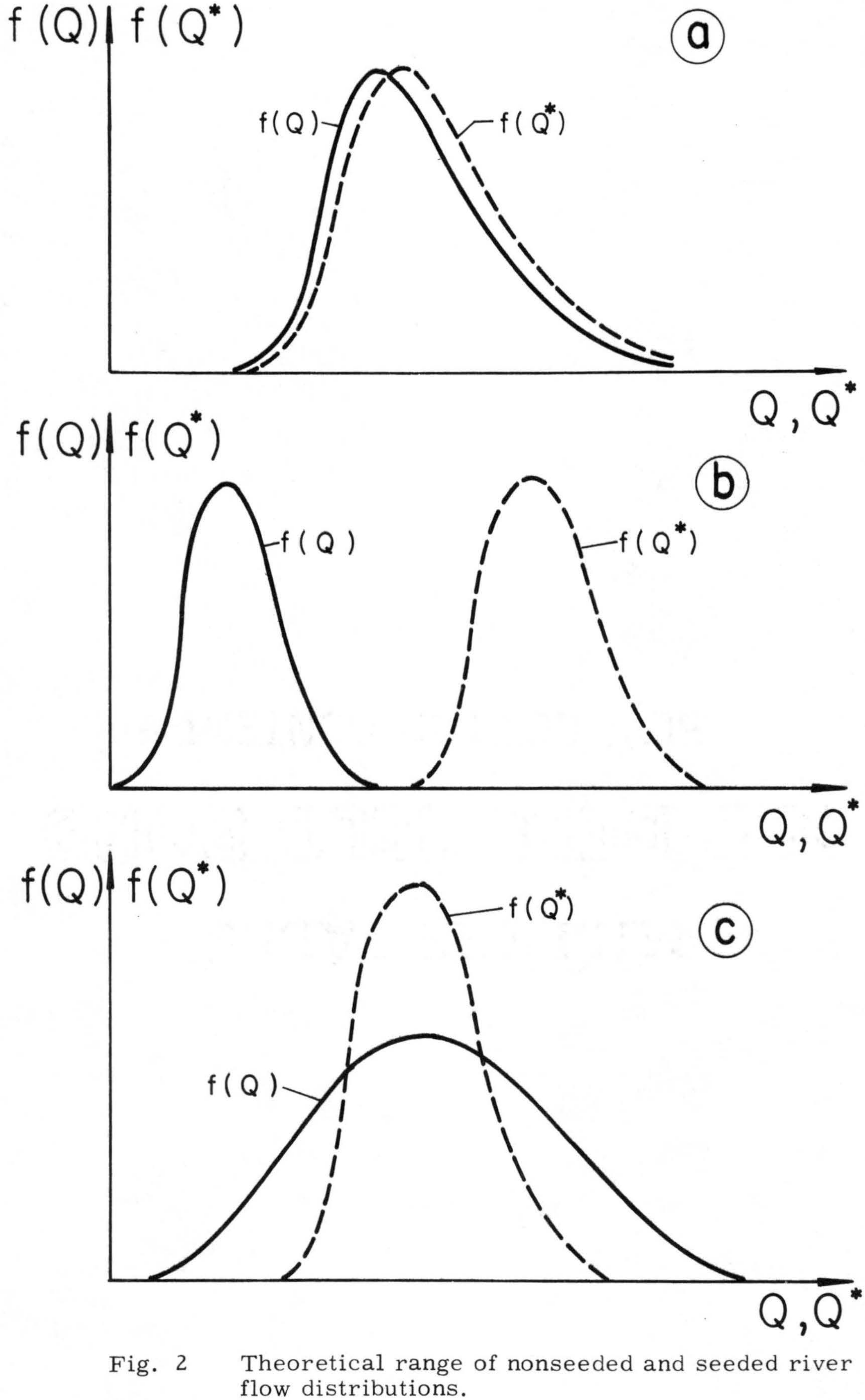

Fig. 2 Theoretical range of nonseeded and seeded river flow distributions.

populations happen to have the same mean or median but with the nonseeded population (Q's) dispersed and the seeded population (Q*'s) concentrated, a long Q run will still occur on each end of the combined sample, thus creating a tendency to reduce the number of runs (Figure 2c).

Obviously, the number of runs can play an important role in the construction of a test. The task, then, is to determine the distribution of a number of runs, η, in order to specify η_α for a given level of significance α (the probability α for the type I error). Once this has been accomplished the working hypotheses can be tested. Since the main goal of cloud seeding operations is to increase water yield, the effect of these operations should logically be tested by utilizing the theory of runs under the following null and alternative hypotheses:

H_o: There is no increase in annual river flows of the target basin, and

H_a: There is an increase in annual river flows caused by cloud seeding operations.

First, a simple result on the distribution of arrangements of two sets of annual observations of river flows from the same target population should be obtained. Any arrangement, representing a single sample point, is a sequence of ordered nonseeded (Q's) and seeded (Q*'s) annual river flows consisting of alternating runs of Q's and Q*'s. The set of all these sample points or arrangements constitutes the sample space. Now, this sample space, its sample points consisting of the set of all possible arrangements or combinations of n Q's and m Q*'s,

$$N\{S\} = \binom{n+m}{n} \qquad (2)$$

should be considered. All these arrangements are equally likely under the null hypothesis. Next, it is necessary to count all the arrangements with exactly η runs. If η is assumed to be even, then there must be $\eta/2$ runs of Q's and $\eta/2$ runs of Q*'s. To find the $\eta/2$ runs of Q's, the n Q's must be divided into $\eta/2$ groups, and all $\eta/2$ numbers in each group must be counted. Then, the required number, the ordered $\eta/2$ - part partitions on n with zero parts excluded, can be found by means of the combinatorial generating function as the coefficient of t^n in the following identity [Mood and Graybill, 1963]:

$$\begin{aligned}(t + t^2 + t^3 + \ldots)^{\eta/2} &\equiv t^{\eta/2} \left(\frac{1}{1-t}\right)^{\eta/2} \\ &= t^{\eta/2}\left[1 + \binom{\eta/2}{1} t + \binom{\eta/2+1}{2} t^2 + \right. \\ &\quad \left. + \binom{\eta/2+2}{3} t^3 + \ldots\right] \\ &= t^{\eta/2} \sum_{i=1}^{\infty} \binom{\eta/2 - 1 + i}{\eta/2 - 1} t^i \qquad (3)\end{aligned}$$

which is $\binom{n-1}{\eta/2-1}$. Similarly, there are $\binom{m-1}{\eta/2-1}$ $\eta/2$ - part partitions of m, excluding zero parts. Now, any partition of Q's may be combined in any partition of Q*'s in two ways to form a sequence as in (1): the first Q partition or the first Q* partition may be put at the beginning of the sequence. There are, therefore,

$$N\{E\} = 2\binom{n-1}{\eta/2-1}\binom{m-1}{\eta/2-1} \qquad (4)$$

arrangements with exactly η runs. Thus, the probability function for even values of η is:

$$p(\eta) = \frac{N\{E\}}{N\{S\}} = \frac{2\binom{n-1}{\eta/2-1}\binom{m-1}{\eta/2-1}}{\binom{n+m}{n}} \tag{5}$$

A similar result will hold, of course, for the probability density function for odd values of η [Wilks, 1962],

$$p(\eta) = \frac{\binom{n-1}{\eta/2}\binom{m-1}{\eta/2-1} + \binom{n-1}{\eta/2-1}\binom{m-1}{\eta/2}}{\binom{n+m}{n}} \tag{6}$$

To test the null hypothesis at the α level of significance, the critical value of η, η_α must be found from the probability that the number of runs is equal to or less than η_α in a random arrangement; this probability is given by:

$$P\{\eta \leq \eta_\alpha\} = \sum_{\eta=2}^{\eta_\alpha} p(\eta) = \alpha \tag{7}$$

The test is then performed by observing the total number of runs in the combined nonseeded-seeded sample of $n + m$ observations; the null hypothesis is accepted if the observed number of runs, η_o, is greater than the specified (critical) number of runs at the α level of significance, η_α, or the null hypothesis is rejected if the observed number of runs does not exceed the critical number of runs. In other words, if

$$\eta_o \geq \eta_\alpha \tag{8}$$

there is no significant difference between the nonseeded and seeded annual river flows (Figure 3); if there is, the reverse is true.

The computation of equation (7) can be quite involved unless both n and m are small. The distribution of η becomes approximately normal for large samples, and in fact, the approximation is usually good enough for practical purposes when both n and m exceed 10. The mean of the distribution of runs is then:

$$E(\eta) = \frac{2nm}{n+m} + 1 \tag{9}$$

and the variance is:

$$\sigma^2_\eta = \frac{2nm\,(2nm - n - m)}{(n+m)^2\,(n+m-1)} \tag{10}$$

By use of the unit normal deviate,

$$u = \frac{\eta - E(\eta)}{\sigma_\eta} \tag{11}$$

η_α can be determined for testing the null hypothesis at the α level of significance. This is done by making the right-hand side of equation (10) equal to the critical value of u for the same level of significance of the normal function and by solving for η:

$$\eta_\alpha = u_\alpha\, \sigma_\eta + E(\eta) \tag{12}$$

The two-sample run test should be sensitive to both differences in shape and in mean between the nonseeded and seeded annual river flow distributions.

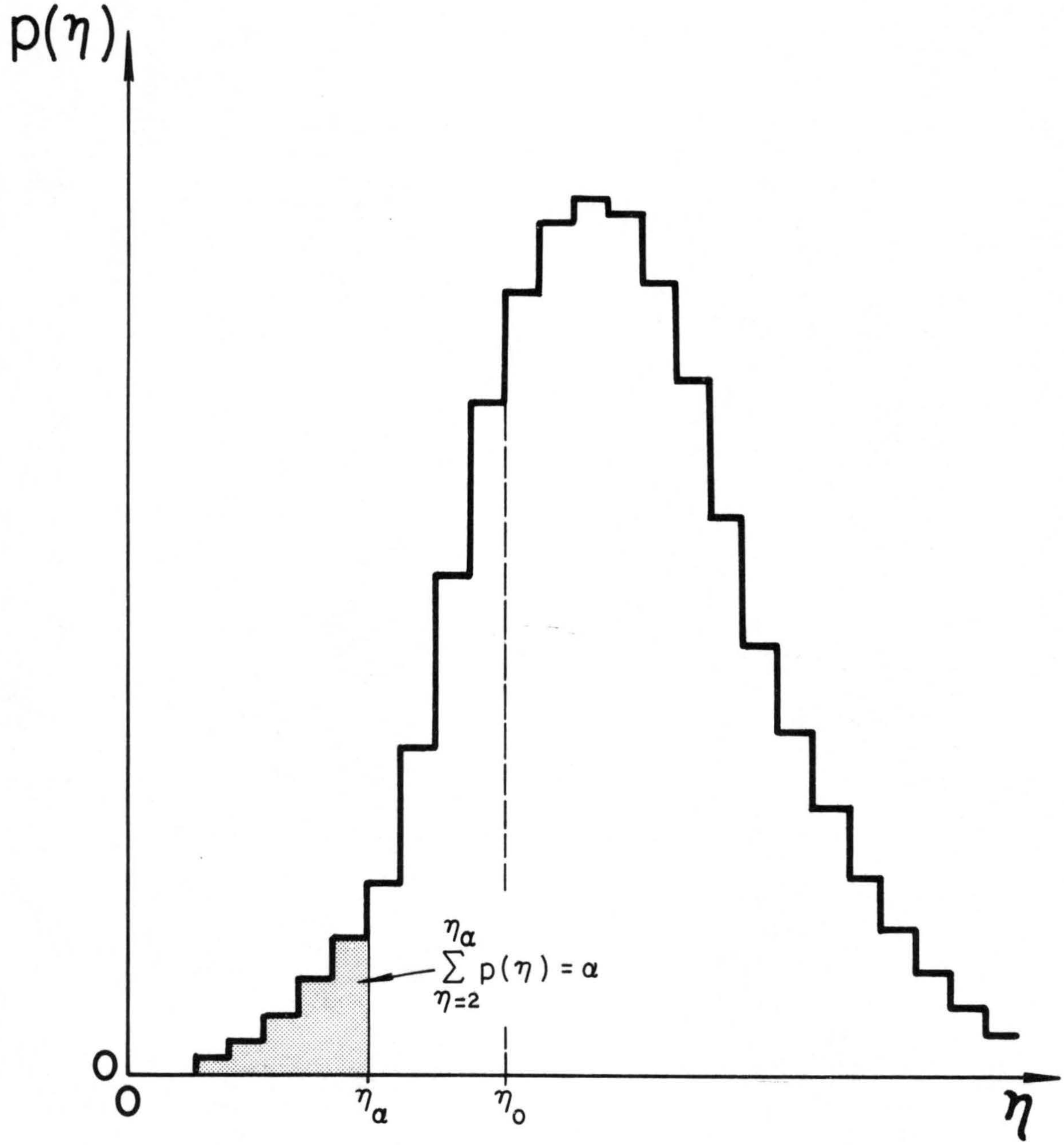

Fig. 3 Frequency histogram of the number of runs of nonseeded and seeded river flows.

PRACTICAL APPLICATION

Essentially, the purpose of this paper is to illustrate the application of method rather than to evaluate a particular weather modification project. However, all weather modification projects are not equally suitable, even for illustration purposes. Generally, for evaluation purposes, the river basin subjected to weather modification experiments should, as nearly as possible, meet the following conditions:

(1) Gaged river basin, preferably equipped with recorders;

(2) Long period of river flow observations prior to weather modification experiments;

(3) Long period of river flow observations during weather modification experiments;

(4) Accurate and reliable data, preferably classified excellent or very good;

(5) Continuous and uniform weather modification experiments over entire river basin (i. e., not partial or random);

(6) Unchanged natural conditions of river basin in both non-seeded and seeded period (i. e., no nonhomogeneity in river flow data), and preferably no diversions or storage in reservoirs.

The above conditions are exacting, and very few river basins can fully satisfy the majority of them. Nevertheless, a survey of past and present weather modification projects indicated that the Kings River Basin in California can, at least partially, fulfill the majority of them. Here a project rare in the United States and in the rest of the world was carried out. This river basin has been solely and continuously treated by one unique weather modification technique--cloud seeding with silver iodide--for more than twelve years. In addition, river flow data is available for nonseeded as well as for seeded periods at several gaging sites registering the runoff from upstream drainage areas. From these several gaging sites, the data from the Kings River at Piedra, California, are used to demonstrate the application of the target two-sample run method of statistical evaluation of weather modification.

The observed river flows used here are those from the nonseeded period, 1917 to 1954, and from the seeded period, 1955 to 1966 water year. These $n = 38$ nonseeded and $m = 12$ seeded annual river flows of the Kings River at Piedra are listed in Table 1 and graphed in Figure 4.

From Table 1 and Figure 4 it is obvious that the observed number of runs is:

$$\eta_o = 17 .$$

Then, from equation (9), the mean of the distribution can be determined:

$$E(\eta) = \frac{2 \times 38 \times 12}{38 + 12} + 1 = 19.240$$

And, using equation (10), the variance of the distribution is:

$$\sigma^2_\eta = \frac{2 \times 38 \times 12 \, (2 \times 38 \times 12 - 38 - 12)}{(38 + 12)^2 \, (38 + 12 - 1)} = 6.417$$

Therefore, the following is standard deviation of the distribution:

$$\sigma_\eta = \sqrt{6.417} = 2.533 .$$

Finally, according to equation (12), the critical value of η at the α

Table 1. Annual river flows of the Kings River at Piedra, California.

i	Year	Q_i (m^3/sec)	$Q_{(i)}$ (m^3/sec)	$Q_{(i)}$ & $Q^*_{(j)}$ (m^3/sec)
1	1917	73.907	15.263	15.263
2	8	53.236	18.236	18.236
3	9	47.006	25.740	22.370*
4	1920	54.652	33.131	25.740
5	1	60.032	33.697	28.090*
6	2	85.801	37.577	32.225*
7	3	60.882	37.945	33.131
8	4	15.263	38.115	33.697
9	1925	50.404	38.823	34.320*
10	6	40.493	40.493	37.577
11	7	77.589	43.325	37.945
12	8	37.577	45.166	38.115
13	9	33.131	45.562	38.823
14	1930	33.697	46.157	40.493
15	1	18.236	47.006	43.325
16	2	81.270	50.093	44.713*
17	3	46.157	50.404	45.166
18	4	25.740	52.358	45.562
19	1935	63.402	53.236	46.157
20	6	73.200	54.652	47.006
21	7	91.549	60.032	47.686*
22	8	128.078	60.882	49.243*
23	9	38.115	62.609	50.093
24	1940	69.830	63.062	50.404
25	1	99.449	63.402	52.358
26	2	78.438	69.830	53.236
27	3	79.259	73.200	54.652
28	4	45.562	73.907	60.032
29	1945	80.675	77.589	60.882
30	6	63.062	78.438	62.609
31	7	43.325	79.259	63.062
32	8	38.823	80.675	63.402
33	9	37.577	81.270	69.830
34	1950	50.093	85.801	73.200
35	1	62.609	91.549	73.200*
36	2	111.399	99.449	73.907
37	3	45.166	111.399	74.361*
38	1954	52.358	128.078	77.475*

j	Year	Q^*_j	$Q^*_{(j)}$	$Q_{(i)}$ & $Q^*_{(j)}$
1	1955	44.713	22.370	77.589
2	6	105.113	28.090	78.438
3	7	49.243	32.225	79.259
4	8	102.281	34.320	80.675
5	9	32.225	44.713	81.270
6	1960	28.090	47.686	85.801
7	1	22.370	49.243	91.549
8	2	73.200	73.200	99.449
9	3	74.361	74.361	102.281*
10	4	34.320	77.475	105.113*
11	1965	77.475	102.281	111.399
12	6	47.686	105.113	128.078

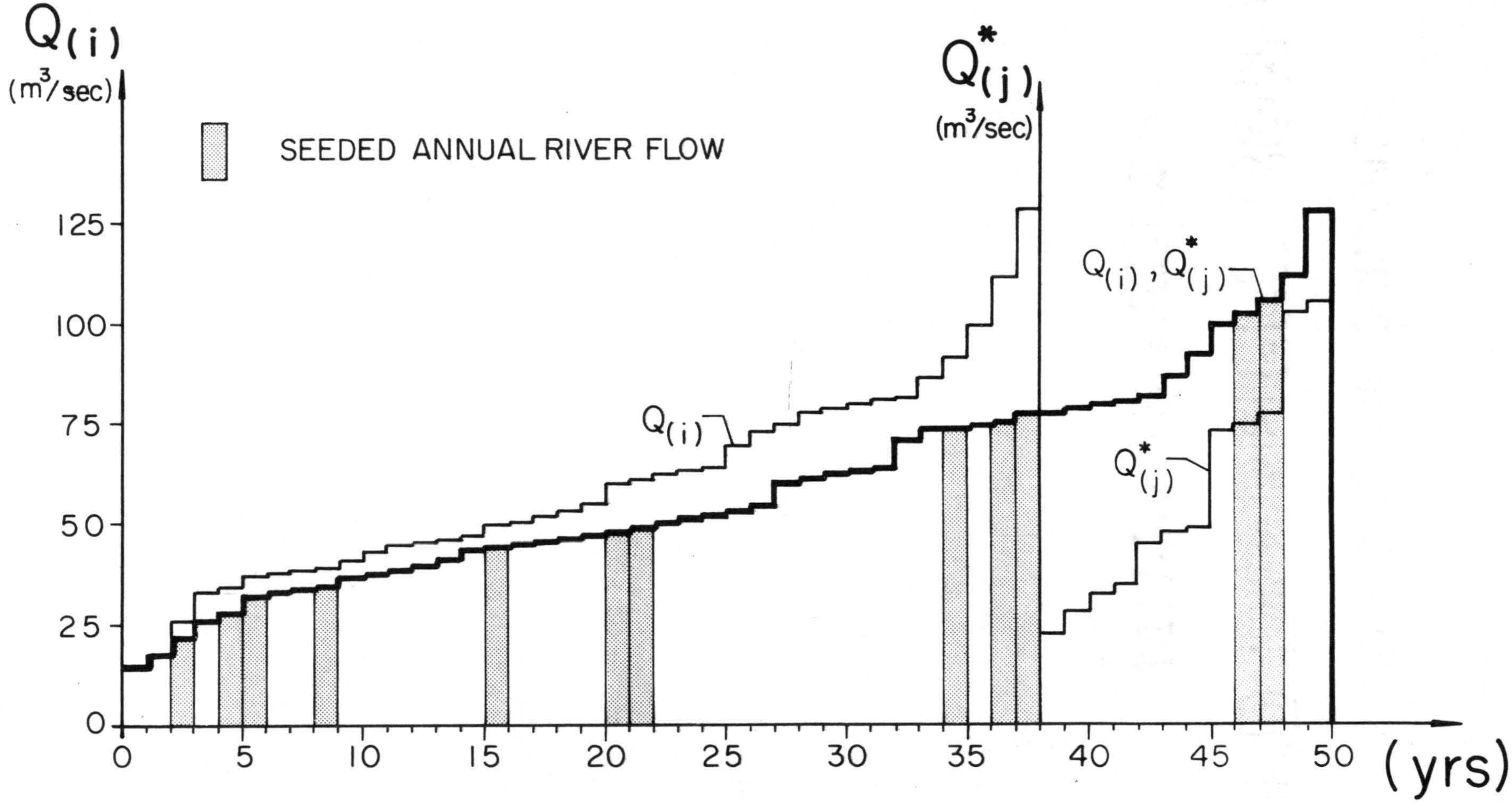

Fig. 4 Duration curves of nonseeded [$Q_{(i)}$], seeded [$Q^*_{(j)}$], and combined [$Q_{(i)}$, $Q^*_{(j)}$] period annual river flows of the Kings River at Piedra, California.

level of significance can be determined to be:

$$\eta_\alpha = -1.645 \times 2.533 + 19.240 = 15.073 \approx 15 .$$

The test of significance, preformed in accordance with equation (8),

$$\eta_o = 17 > \eta_{0.05} = 15$$

indicates that the null hypothesis can be accepted at the 5 per cent level of significance. In other words, there is no statistically significant change in annual river flows of the target basin caused by cloud seeding operations.

Although the test result is statistically nonsignificant, the small difference between the observed and the critical number of runs indicates that the target two-sample run test is sensitive to changes in river flow distributions. It also indicates that cloud seeding operations may produce a certain change in river flows but that such change is within the range of the natural fluctuations of annual river flow distributions.

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